Energetic Macroscopic Representation and Inversion-based Control: Application to an Electric Vehicle with an Electrical Differential

Keyu Chen [1], **Alain Bouscayrol** [2], **and Walter Lhomme** [3]

[1] *Laboratoire d'Electrotechnique et Electronique de Puissance, University of Science and Technology of Lille, chenkeyucky@yahoo.fr*
[2] *Laboratoire d'Electrotechnique et Electronique de Puissance, University of Science and Technology of Lille, Alain.Bouscayrol@univ-lille1.fr*
[3] *Laboratoire d'Electrotechnique et Electronique de Puissance, University of Science and Technology of Lille, walterlhomme@yahoo.fr*

Abstract

Energetic Macroscopic Representation (EMR) is an energy-based graphical modelling tool to describe complex electromechanical systems. It is based on the action-reaction principle to organize the interconnection of sub-systems according to the physical causality (i.e. integral causality). Moreover, an inversion-based control can be systematically deduced from EMR using specific inversion rules. The aim of this paper is to introduce the basics of EMR approach and its inversion-based control. An Electric Vehicle with an electrical differential is studied as a simple example.

Keywords

energetic macroscopic representation, electric vehicle, electrical differential, control

1. INTRODUCTION

With the rapid development of science and technology, emergence of new complex systems has never ceased. How to organize the modelling and control of these systems becomes more and more like a challenge. Especially, for energetic systems, the considerations of energy should be emphasized. Thus since 1950s, Bond Graph has been introduced and developed by H. M. Paynter of MIT and his team [Paynter, 1961]. Bond Graph contributes significantly to the modelling of energetic systems but it can not help the control design directly. In 1996, Causal Ordering Graph (COG) has been introduced to describe power electronics and electrical machines for developing their control [Hautier, 2004]. In 2000, based on COG, Energetic Macroscopic Representation (EMR) has been introduced to describe complex electromechanical systems [Bouscayrol, 2000]. Same as Bond Graph and COG, EMR is an energy-based graphical tool but has a more global energetic view. Differing from Bond Graph, EMR is focused on the system function but not only on the system structure. Therefore EMR gives insights into the real energy operation of system and allows a deep understanding of its potentialities from a dynamic point of view. Due to these features of EMR, Inversion-based control can be deduced from EMR [Hautier, 2004].

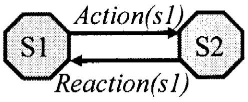
In this paper, the EMR approach and its inversion-based control will be introduced. An Electric Vehicle (EV) with an electrical differential will be used as an example.

2. EMR FUNDAMENTALS

EMR is based on the action-reaction principle to organize the interconnection of sub-systems according to the physical causality (i.e. integral causality). This description highlights energetic properties of the system (energy accumulation, conversion and distribution).

2.1 Action-reaction principle

Power is transmitted between connected elements by a combination of "action" and "reaction". For example, a current source (s1) is connected to a capacitor (s2), the action of s1 is its current and the voltage is its reaction by s2. The transmitted power is the product of the action and the reaction.



Power = Action × Reaction

Fig. 1 Action-reaction principle

2.2 Integral causality

In a real time system, integration means the surface of the past but derivation means the trend of the future which can not be determinate before [Iwasaki, 1994]. Thereby, EMR considers integral causality as the only allowable physical causality. Integral causality indicates what are the inputs of system and what are the outputs. In other words, for a system modeled by an integral equation, integrand (cause) is the input of system and the integra-

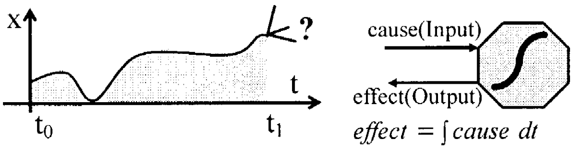


Fig. 2 Integral causality principle

tion (effect) is the output. For instance, as viewed from the capacitor equation, the current is the cause (input) and the voltage is the effect (output). Contrarily to an inductor. A resistor has no time-dependant behavior: voltage or current can be considered as input or output, that depends on the system connected with it.

2.3 Elements

EMR has 4 basic kinds of elements: source elements, conversion elements, accumulation elements and coupling elements (Table 1). Source elements (ovals) are terminal elements, which deliver or receive energy (e.g. a battery). Conversion elements are considered as energy conversion without energy accumulation. They are described by squares for electrical conversion, circles for electromechanical conversion and triangles for mechanical conversion (e.g. Table 2: Example 1 - a transformer). Accumulation elements (rectangles) store energy, which leads to state variables. In Table 2: Example 2 - an inductor, since the integrand (cause) is the difference of v_1 and v_2, so the inputs are the two voltages. The current i is the effect, so i is the output, namely the state variable of system. Coupling elements (overlapped pictograms) are introduced to distribute energy (e.g. series or parallel relationship of electrical circuits).

2.4 Inversion-based control

The inversion based control theory has been initiated by COG [Hautier, 2004]: The control structure of a system is considered as an inversion model of the system. Firstly, tuning chains are defined according to the objective and

Table 1 EMR elements, control blocks and examples

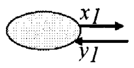
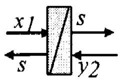
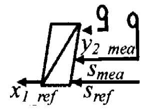
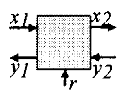
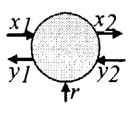
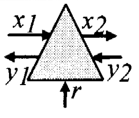
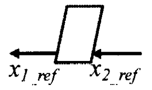
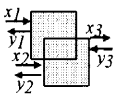
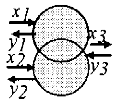
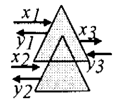
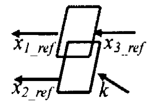
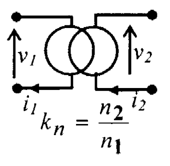
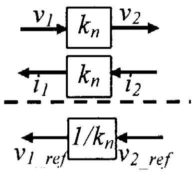
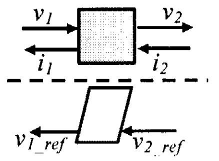
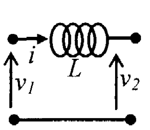
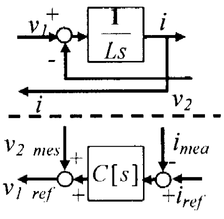
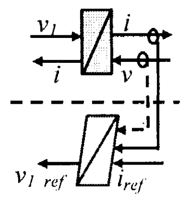
Source Elements				**Accumulation Elements**		**Control block**
						
Conversion Elements						**Control block**
Electrical Conversion		Electro -mechanical Conversion		Mechanical Conversion		
Coupling Elements						**Control block**
Electrical Coupling		Electro -mechanical Coupling		Mechanical Coupling		

Table 2 Examples -

	System	Equations	Block diagrams	EMR and inversion-based control
Example 1		$\begin{cases} v_2 = k_n v_1 \\ i_1 = k_n i_2 \end{cases}$ $v_{1\_ref} = \frac{1}{k_n} v_{2\_ref}$		
Example 2		$i = \frac{1}{L}\int(v_1 - v_2)dt$ $v_{1\_ref} = C[i_{ref} - i_{mea}] + v_{2\_ref}$		

constraints of system. Then, control chains are deduced by inversions of the tuning chains. According to these control chains, EMR is inverted element by element. All control blocks are depicted by parallelograms as they handle only information (Table 1). Conversion elements (squares, circles and triangles) can be directly inverted, as they have no time-dependence behaviour. Coupling elements (overlapped pictograms) may require supplementary inputs k for inversion. Since accumulation elements (rectangle with an oblique bar) cannot be inverted physically to avoid derivation, an indirect inversion is thus made using a controller. Some examples of EMR elements and its inversion-based control blocks are compared with block diagrams in Table 2. In example 1, if the chosen tuning chain is $v_1 \rightarrow v_2$ for a transformer, the control chain is $v_{1\_ref} \leftarrow v_{2\_ref}$. Then this EMR element can be inversed directly . In example 2, if the chosen tuning chain is $v_1 \rightarrow i$ for an inductor, the control chain is $v_{1\_ref} \leftarrow i_{\_ref}$. Since an accumulation element represents a time-dependence relationship, a controller is required. Small circles mean sensors. Real arrow line with sensor means that this measurement is necessary. Dashed arrow line with sensor means the disturb compensation which should be taken into account in the fist step. This operation can be simplified during the second step when dealing with the control implementation. By this way, Maximum Control Structure can be obtained under the assumption that all variables are measurable. From this Maximum Control Structure, some simplifications and estimations can be made and then a Practical Control Structure can be obtained.

3. ELECTRIC VEHICLE WITH AN ELECTRICAL DIFFERENTIAL

3.1 Structure of the vehicle traction system

Nowadays, traction schemes with an electrical differential have been suggested [Crelerot, 1993] [Ledezma, 2001]. In this paper, an EV with a electrical differential will be studied as an example. The two front wheels of an electric vehicle are driven by two permanent magnet DC machines (EM1 and EM2) (Figure 3). These machines are fed by two choppers which are supplied by a common battery. There is no mechanical differential and the dual-drive traction system must ensure independent rotation speeds for curves. Mechanical braking is not taken in to account in this paper.

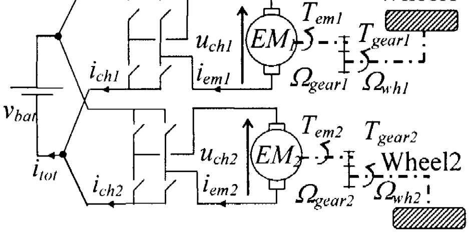


Fig. 3 Traction system scheme

3.2 EMR of the vehicle traction system

The EMR of the system is depicted in the upper part of Figure 4. From left to right, the different parts are described as follows.

(a) Electrical source

The battery (oval) delivers the dc voltage v_{bat} to supply the traction system, which produces a current i_{tot}.

(b) Parallel connection

The parallel connection is a coupling device (overlapped square), which distributes the DC voltage v_{bat} to both choppers. The total current i_{tot} is the sum of the currents of the two choppers i_{chl} and ${}_{ich2}$:

$$\begin{cases} \text{common} \quad v_{bat} \\ i_{tot} = i_{ch1} + i_{ch2} \end{cases} \tag{1}$$

(c) Choppers

The chopper (squares) leads to the chopper voltage $u_{ch(i)}$ from batter voltage v_{bat}, and the chopper current $i_{ch(i)}$ from the machine current $i_{em(i)}$:

$$\begin{cases} u_{ch(i)} = m_{(i)}\, v_{bat} \\ i_{ch(i)} = m_{(i)}\, i_{em(i)} \end{cases} ; \ i \in \{1,2\} \tag{2}$$

with $m_{(i)}$ modulation of the chopper and i number of choppers.

(d) DC machines

The armature winding of DC machine, which accumulates energy (rectangles), leads to the armature current $i_{em(i)}$ as the state variable from the e.m.f. $e_{(i)}$ and the chopper voltage $u_{ch(i)}$:

$$u_{ch(i)} - e_{(i)} L_{arm(i)} = \frac{d}{dt} i_{em(i)} + R_{arm(i)} i_{em(i)} \tag{3}$$

where $L_{arm(i)}$ and $R_{arm(i)}$ are the winding inductance and resistance. The DC machine torque $T_{em(i)}$ is obtained from $i_{em(i)}$. The e.m.f. $e_{(i)}$ is linked to the rotation speed $\Omega_{gear(i)}$:

$$\begin{cases} T_{em(i)} = k_{dcm(i)}\, i_{em(i)} \\ e_{(i)} = k_{dcm(i)}\, \Omega_{gear(i)} \end{cases} \tag{4}$$

where $k_{dcm(i)}$ is the torque coefficient of the electromechanical conversion (circles).

(e) Gears

If the slip phenomenon of the wheels is ignored, all inertia are merged with the vehicle mass. The reduction gear torque $T_{gear(i)}$ and its rotation speed $\Omega_{gear(i)}$ are obtained from the machine torque $T_{em(i)}$ and the rotation speed of the wheel $\Omega_{wh(i)}$ (triangles):

$$\begin{cases} T_{gear(i)} = k_{gear}\, T_{em(i)} \\ \Omega_{gear(i)} = k_{gear}\, \Omega_{wh(i)} \end{cases} \tag{5}$$

where k_{gear} is the reduction gear ration.

(f) Wheels

The wheel traction force $F_{wh(i)}$ is obtained from the gear torque $T_{gear(i)}$ and the wheel rotation speed from the wheel velocity vev using the wheel radius R_{wh} (triangles):

$$\begin{cases} F_{wh(i)} = \dfrac{1}{R_{wh}} T_{em(i)} \\ \Omega_{wh(i)} = \dfrac{1}{R_{wh}} v_{wh(i)} \end{cases} \tag{6}$$

(g) Chassis

A coupling device (double triangle) yields both the total traction force F_{tot} and the wheel velocities $v_{wh(i)}$:

$$\begin{cases} v_{wh1} = \dfrac{R_{curv} + l_{ev}/2}{R_{curv}} v_{ev} \\ v_{wh2} = \dfrac{R_{curv} - l_{ev}/2}{R_{curv}} v_{ev} \\ F_{tot} = (1 + \dfrac{lev/2}{R_{curv}}) F_{wh1} + (1 - \dfrac{lev/2}{R_{curv}}) F_{wh2} \end{cases} \tag{7}$$

where l_{ev} is the width of the vehicle and R_{curv} is the radius of the curve defined by the steering angle. The vehicle speed v_{ev} is obtained by the dynamics relationship:

$$M \frac{d}{dt} v_{ev} = F_{tot} - F_{res} \tag{8}$$

with M the vehicle mass and F_{res} resistant force. This relationship is represented by an accumulation element (rectangle) with the vehicle speed v_{ev} as the state variable.

(h) Mechanical source

The environment is a Mechanical Source (oval) which yields the resistant force F_{res}:

$$F_{res} = F_0 + a\, v_{ev} + b v_{ev}^2 + Mg \sin\alpha, \tag{9}$$

with F_0 the initial rolling force, a the rolling coefficient, b the drag coefficient, α the slope rate and gravity.

3.3 Control structure of the traction system

An inversion-based control is obtained from EMR using inversion rules (lower part of Figure 4). The Maximum Control Structure is presented in this paper. Practical Control Structure can be referred in [Bouscayrol, 2006].

3.3.1 Tuning chain and control chain

There are 2 tuning variables m_1 and m_2 in the system. One objective (control of the vehicle velocity v_{ev}) should be achieved. So there is one degree of freedom which could be used though (11). According to the objective, the tuning chains are chosen:

$$\begin{tikzcd}
m_1 \arrow[r] & u_{ch1} \arrow[r] & i_{em1} \arrow[r] & T_{em1} \arrow[r] & T_{gear1} \arrow[r] & F_{wh1} \arrow[dr] & & \\
 & & & & & & F_{tot} \arrow[r] & v_{ev} \\
m_2 \arrow[r] & u_{ch2} \arrow[r] & i_{em2} \arrow[r] & T_{em2} \arrow[r] & T_{gear2} \arrow[r] & F_{wh2} \arrow[ur] & &
\end{tikzcd}$$

Then control chains are obtained:

$$\begin{tikzcd}
m_1 & & & & & & & \\
u_{ch1\_ref} \arrow[u] & i_{em1\_ref} \arrow[l] & T_{em1\_ref} \arrow[l] & T_{gear1\_ref} \arrow[l] & F_{wh1\_ref} \arrow[l] & & & \\
m_2 & & & & & F_{tot\_ref} \arrow[ul] \arrow[dl] & v_{ev\_ref} \arrow[l] & \\
u_{ch2\_ref} \arrow[u] & i_{em2\_ref} \arrow[l] & T_{em2\_ref} \arrow[l] & T_{gear2\_ref} \arrow[l] & F_{wh2\_ref} \arrow[l] & & &
\end{tikzcd}$$

3.3.2 Maximum control structure

(a) Inversion of chassis

From right to left, the first element is an accumulation element (8). A controller is need to define the traction force $F_{tot\ ref}$ from the vehicle velocity reference v_{ev_ref}

$$F_{tot} = C[v_{ev\_ref} - v_{ev\_mea}] + F_{res\_mea} \tag{10}$$

where C is the controller (PI, IP or other types of controllers). Actually, the velocity controller corre-

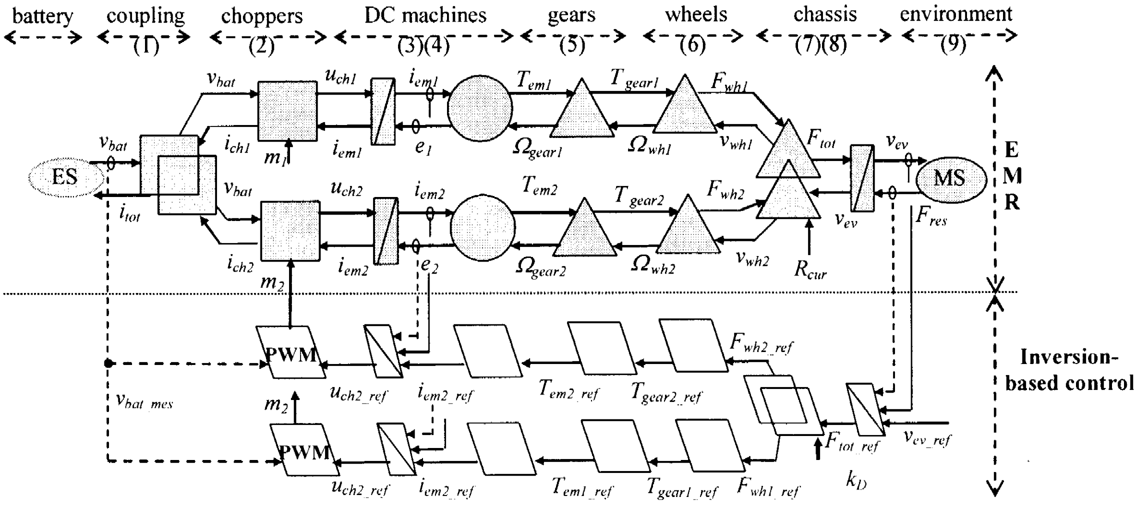


Fig. 4 EMR and control structure of the EV traction system

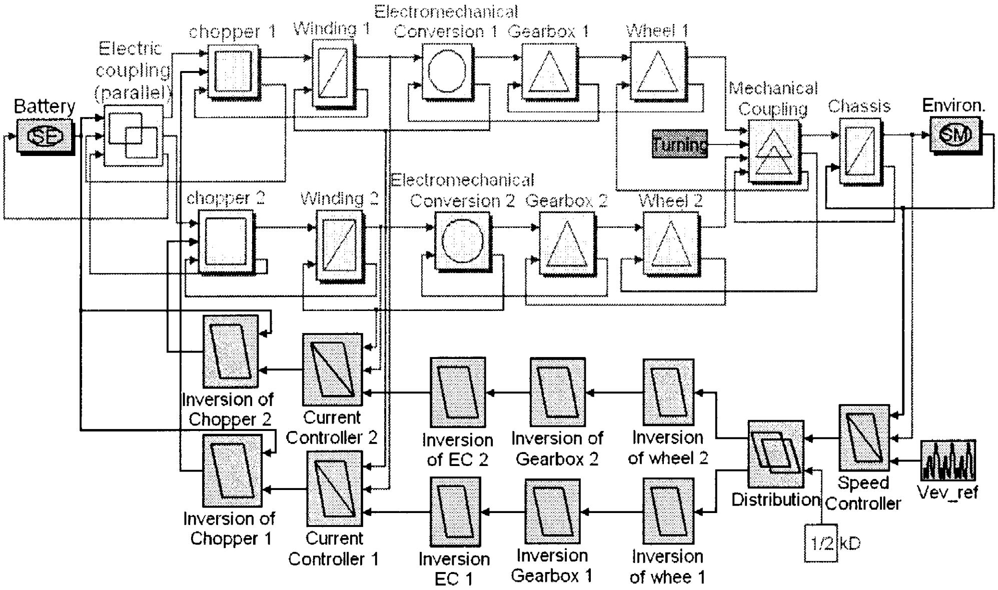


Fig. 5 Simulation using Matlab-Simulinkc

sponds to the behaviour of driver.

(b) Inversion of mechanical coupling

There are 2 inputs and 1 output according to the chosen tuning chain. So a supplementary input k_D is needed to inversion (7):

$$\begin{cases}(\mathbf{1}+\frac{lev/\mathbf{2}}{R_{curv}})F_{r\mathbf{1}}=k_D F_{tot\_ref}\\(\mathbf{1}-\frac{lev/\mathbf{2}}{R_{curv}})F_{r\mathbf{2}}=(\mathbf{1}-k_D)F_{tot\_ref}\end{cases}\text{with } \mathbf{0}\le k_D\le\mathbf{1} \quad (11)$$

In order to ensure an equal distribution of forces, k_D is set to 1/2 as in most traction applications. Thus the remained freedom is used through k_D.

(c) Inversion of wheels

Its time-independent relationship (6) can be directly inverted:

$$T_{gear(i)\_ref}=R_{wh}F_{wh(i)\_ref} \quad (12)$$

(d) Inversion of gears

Relation (5) is directly inverted to obtain the torque reference T_{em_ref} from the gearbox torque reference:

$$T_{em(i)\_ref}=\frac{\mathbf{1}}{k_{gear}}T_{gear(i)\_ref} \quad (13)$$

(e) Inversion of DC machines

The reference current $i_{em(i)\_ref}$ is deduced from a direct inversion of (4):

$$i_{em(i)\_ref}=\frac{\mathbf{1}}{k_{dcm(I)}}T_{em(i)\_ref} \quad (14)$$

As the winding is an accumulation element (3), its inversion requires a controller:

$$u_{ch(i)\_ref}=C\left[i_{em(i)\_ref}-i_{em(i)\_mea}\right]+e_{(i)} \quad (15)$$

The compensation of e.m.f can be estimated.

(f) Inversion of choppers

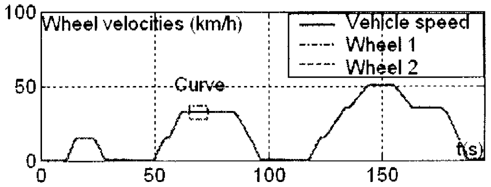
The chopper modulation $m_{ch(i)\_ref}$ is deduced from a direct inversion of (2) with the measurement of the battery voltage v_{bat_mea}:

$$m_{ch(i)}=\frac{\mathbf{1}}{v_{bat\_mea}}u_{ch(i)\_ref} \quad (16)$$

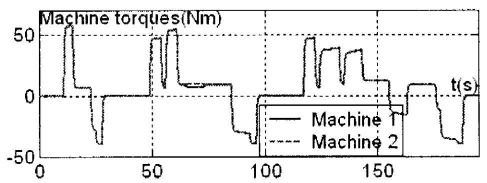
These continuous modulation functions are converted to discrete variables using a PWM.

3.4 Simulation results

By the help of EMR, some simulation are made using Matlab-Simulink© (Figure 5). Under an Urban Driving Cycle with a curve at t = 65 s for 8 s, the vehicle speed, the velocities of the two traction wheels and two DC machine torques are showed in Figure 6. As k_D is set to



(a)Vehicle speed and Wheel velocities



(b) Machine torques

Fig. 6 Simulation results

1/2, the two wheels have the same velocity and the two driven machines have the same torque in straight line. When the vehicles turns, the external wheel of the curve is faster than the internal wheel and the machine torque of the external wheel is weaker than internal one's. Generally, supplementary inputs are used to the strategies of system [Verhille, 2004], for example to optimiser the energy distribution.

4. CONCLUSION

The basics of EMR approach and its inversion-based control is presented in this paper. The EMR and control structure of an electric Vehicle with an electrical differential is introduced as a simple example. More complex principles can be found in [Lhomme, 2008]. The distinct features of EMR lie in its clarity of physical concepts, as well as their integral causality, and its functional modelling rather than a structural modelling, hence contribute significantly to the design of the control and to energy management of systems.

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