

# Development of a system to recommend piano practice music utilizing difficulty level estimation algorithm

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## Abstract

This study aims to develop a system to recommend an adequate difficulty level of music which suits a preference of a self-learning piano student. First, a user input a group of favorite music into a system and the system arranges the music according to the difficulty level. Next, the system analyzes the proficiency level of the user and recommends the music with the difficulty which is most similar to this level. With this system, users can select their practice music from the group of music arranged according to the difficulty level. In this paper, the authors used following procedure: first, a group of music from a piano manual which arranged several pieces of music according to the difficulty level are registered into a system; next, the system selects a piece of music based on the preference and the proficiency level of the user. The proficiency level is judged by whether the user can play the selected music or not. We proposed two algorithms to estimate the similarity of the difficulty level of music: One employs nearest neighbor algorithm and the other utilizes neural network.

## Key words

difficulty level estimation, piano music, self-learning, nearest neighbor algorithm, neural network

## 1. Introduction

In Japan, piano lessons are quite popular for children's education outside the school. According to Suenaga (2008), among elementary and junior high students who experienced non-school related lessons, those who practiced piano was highest with 29 % rate of experience.

Recently, the number of adults who start to practice the piano after they graduated from school is growing. A company which runs music schools nationwide offers piano lessons for adults in 1,300 locations (Yamaha Music Foundation). In order to improve piano proficiency, taking these lessons could be effective. However, a time constraint from work or house chore make it nearly impossible to take these lessons, hence many adults are forced to learn piano by themselves. There are many challenges for piano self-learners; they can't get objective evaluation from a third person such as a piano instructor; they have to manage practice by themselves; and they have to select music for their practice. This study aims to propose a solution to one of the challenges which enables a learner to select music appropriate to his/her proficiency.

Two aspects are important to select music: One is the fitness with learners' preference and the other is the adequateness of the difficulty level. If the selected music fits the learner's preference but the difficulty level is too high, there is a high probability that he/she will find it difficult to practice. On the other hand, if the difficulty level is adequate to the learner but doesn't fit his/her preference, the learner will lose

the motivation to practice. In either case, there is a high probability that the learner will stop practicing halfway. Thus, it is highly important to select music which matches the learner's preference and the proficiency.

Currently, the authors are not able to find a study which estimates the difficulty level of a whole piece of music. And there are no proposal for a procedure to arrange a given group of music according to the difficulty level. There are, however, two similar studies; one (Miyagawa, 2003) investigates a difficult part of music to play and the other (Kido et al., 2003) analyzes the difficulty level of a piano manual called *Beyer*.

Miyagawa (2003) performed multiple regression analysis based on information obtained from a piano practice manual called *Burgmuller 25* and parts of the music which an experienced pianist finds it difficult to play and derived evaluation functions to estimate the difficulty level of the scores. Concerning the information from the scores, a number of attributes related to the difficulty level, such as tone difference and the length of each notes, are taken into account.

Kido et al. (2003) tried to find out whether music scores in *Bayer*, a piano beginner's manual, are arranged according to the difficulty level. They used three attributes related to the difficulty level: tone, hands' position, and the tempo. They did not analyze the difficulty of each score but the manual as a whole.

The existing studies have problems concerning the balance of the attributes taken into account. Specifically, there is only one attribute on fingering, which could significantly affect the difficulty level, and the attribute determinant is whether or not finger number is annotated to the note. On the other

hand, three attributes on black keys are taken into account: accidental notations, black keys and black keys without accidental notation. This imbalance needs to be addressed to improve attributes used for difficulty level estimation.

Followings are explanations for fingering, present/absent of finger number, black keys, and accidental notation:

- Fingering: movement of finger during playing the piano. The existing studies take into account passing under, crossing over, and playing the same tone with different fingers as attributes.
- Finger number: a number annotated above or below a note to indicate which finger to use for smooth fingering.
- Black key: within two types of keys in the piano keyboard, the black ones which are placed farther away from a player.
- Accidental notation: a symbol annotated to the left of a note to indicate a change of tone. The symbols include sharp (to raise the pitch in half chromatic tone) and flat (to lower the pitch in half chromatic tone).
- Black key without accidental notation: a score with accidental notation at the beginning or within the same bar.

This study aims specifically to develop a system which recommends music appropriate for a learner's proficiency from a given group of music which fits his/her preference. In order to develop the system, we propose a following procedure: first, a group of music from a piano manual which arranges several pieces of music according to the difficulty level are registered into the system; next, the system selects music based on the preference and the proficiency level of the user.

In order to develop the system, learner's proficiency and the difficulty level of music must be defined. The former can be estimated from the difficulty level of music he/she can play. When the order of the difficulty level is defined, it is possible to develop a system as shown in Figure 1: a music group composed of music A to D which suit the learner's preference is input into the system; when his/her proficiency level is at the difficulty level of music A, the system recommends music C to the learner for practice.

We define the difficulty level of music by calculating the cost of the score. To calculate the cost, we use three attribute sets listed in 2.1, 2.2 and 3.3.1.

We define the proficiency level of the player by using the

difficulty level defined above. If a player can play the score with a certain difficulty level (e.g. music A in Figure 1), the player's proficiency level is defined as equivalent to the difficulty level of music A.

We suppose that a learner has sufficient information for his/her music preference and can choose practice music which matches the preference.

## 2. Proposed procedure

We propose two attribute sets necessary to arrange several pieces of music in a given music group according to the difficulty level. We also propose two algorithms to perform the arrangement.

### 2.1 First attribute set

In order to reduce the imbalance among attributes, we revised several fingering attributes compared to those of the existing studies. Specifically, we take into account fingerings on all notes including notes without finger numbers. For the fingerings, we determined them under the instruction of a piano instructor. The attributes included in the first set are as follows:

- Attribute on black key: the mean value of the ratio of notes with accidental annotations, notes played with black keys, and notes played with black keys without accidental annotations. This attribute combines three attributes on black keys used in the existing studies.
- Length of the note: the mean value and the standard deviation value of the length of the notes in the score.
- Passing under and Crossing over: the ratio of the notes played with passing under or crossing over to all the notes in the score.
- Same notes played with different fingering: the ratio of the same notes played with different fingerings to all the notes in the score.
- Different notes played with same fingering: the ratio of the different notes played with same fingerings to all the notes in the score.
- Tone difference: the mean value and the standard deviation value of the tone difference in the score.
- Change of rhythm: the ratio of the change of notes accompanied with change of rhythm to all the change of

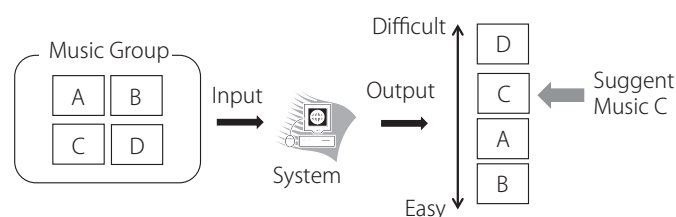


Figure 1: An image of the system with music A as the learner's proficiency

notes in the score.

- Black key: the ratio of the notes played with black key to all the notes in the score.
- Chord: the ratio of the notes composing chords to all the notes in the score.
- Rest: the ratio of the rests in the score.

## 2.2 Second attribute set

In the second set, the attributes included in the first set are aggregated into following two attributes: one reflecting the ease of fingerings and the other reflecting the changes of rhythm. For fingerings, same as the first set, all the notes are taken into account.

### 2.2.1 Attribute reflecting the ease of fingerings

This attribute is defined by the rate of the costs in the music. The cost is evaluated in seven-point scale (from 0 to 6) according to the ease of the fingering between two consecutive notes. <sup>(1)</sup>

Note transition costs are calculated according to the tone difference between two consecutive notes, fingerings, the key of the first note played, present/absent of passing under and/or crossing over. The costs are listed in the cost charts in the appendix. Figure 2 shows an example of a transition cost. For the notes composing a chord, the transition cost is calculated by the mean scores (rounding off one decimal place) of each note in the chord.

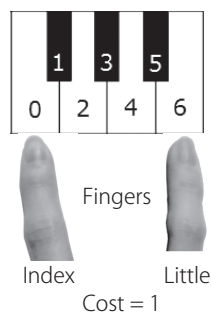


Figure 2: An example of note transition cost

### 2.2.2 Attribute reflecting the change of rhythm

The piano instructor we consulted pointed out in a questionnaire that the music with short notes and frequent rhythm changes tends to be difficult to play because a player finds it difficult to get along with the rhythm. So, in this study, we pick up all the notes with the length of less than 0.5 seconds in the score and calculate the standard deviation of the length of these notes to define the attribute reflecting the change of rhythm.

## 2.3 Algorithms

We propose following two algorithms to arrange several

pieces of music in a given music group according to the difficulty level. One is the nearest neighbor algorithm and the other is algorithm using neural network.

### 2.3.1 The nearest neighbor algorithm

The nearest neighbor algorithm is used for pattern recognition. The algorithm classifies an object by finding the nearest neighbor to assign the object to the class of that single nearest neighbor (Takahashi, 2008). The nearest neighbor algorithm can employ various measurements of the distance such as Euclidean distance and Mahalanobis distance. Our study employs Mahalanobis distance and defines the similarity of the difficulty level among several pieces of music by the nearest distance. Following is the nearest neighbor algorithm we employ to arrange several pieces of music in a given music group according to the difficulty level:

- Algorithm 1
  - (1) Register into a system a group of music  $P_1, P_2, \dots, P_m$  which are arranged in descending order of the known difficulty level in advance of an experiment.
  - (2) Input into the system piano music  $U_1, U_2, \dots, U_t$  ( $m \gg t$ ) which matches user's preference.
  - (3) For  $i = 1, 2, \dots, t$ , calculate the Mahalanobis distance between  $U_i$  and  $P_j$  as  $M(U_i, P_j)$  ( $j = 1, 2, \dots, m$ ) and
    - $M(U_i, P_k) = \text{Min}\{M(U_i, P_1), M(U_i, P_2), \dots, M(U_i, P_m)\}$
    - $A(i) \leftarrow k$ $M(U_i, P_j)$  is a Mahalanobis distance in  $n$ -dimensional space ( $n$ : the number of attributes) using attributes of the music as the dimensions.
  - (4) Arrange  $A(1), A(2), \dots, A(t)$  in the ascending order of size and the result is expressed as  $A(r1), A(r2), \dots, A(rt)$ .
  - (5) The system outputs a series of music which match the user's preference according to the difficulty level as  $U_{r1}, U_{r2}, \dots, U_{rt}$ .

- Example 1

Figure 3 shows an example of the algorithm 1.

### 2.3.2 The algorithm using neural network

The neural network is a mathematical model emulating the information process mechanism of cranial nervous system. It achieves necessary information processing through learning based on given data (Hagiwara, 2006). Neural network is expressed as nodes of computing elements called units. It has been applied in a wide range of fields including pattern categorization, extracting regularity, time sequence analysis and prediction, and data analysis. Our study employs hierarchical neural network to experiment the system we propose. The number of the input layer unit is the number of the attributes. The number of the output layer unit is the number of the music pieces included in the music group arranged

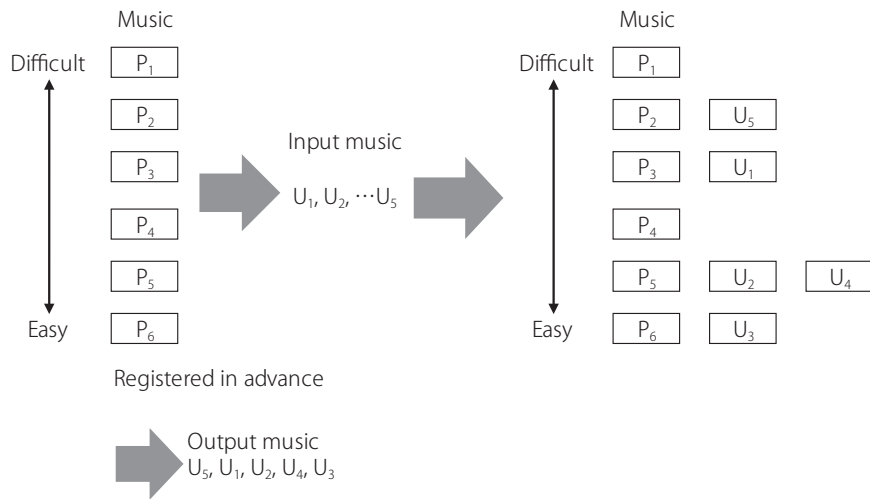


Figure 3: Example of the algorithm 1

according to the difficulty level. Following is the algorithm to determine the closest similarity in difficulty level between a given piece of music and other pieces of music in the music group arranged according to the known difficulty level:

• Algorithm 2

- (1) For the group of music  $P_1, P_2, \dots, P_m$  arranged in descending order of difficulty level, the system learns with the value of input layer as an attribute and the value of output layer as a teachers' signal.
- (2) A given piece of music is used as a test data and the system outputs the music with maximum unit value in output layer.

When the algorithm 2 is performed on all the given music, music in the given music group can be arranged according to the difficulty level.

• Example 2

Figure 4 shows the example of the algorithm 2.

3. Experiment

3.1 Experiment overview

Our experiment verifies the possibility of accurately locating music similar to the difficulty level, with the prerequisite that music in a given group are arranged according to the known difficulty level. Specifically, as shown in Figure 5, we use one music as a test data and find out another music with the nearest difficulty level. If the difficulty level of the music located is contiguous to that of the test data, we consider the result as accurate; if not, we consider it as inaccurate.

3.2 Music used in the experiment

We use 10 music from *Burgmüller's 25 studies for piano*, a piano manual for beginners. Every music in this manual has

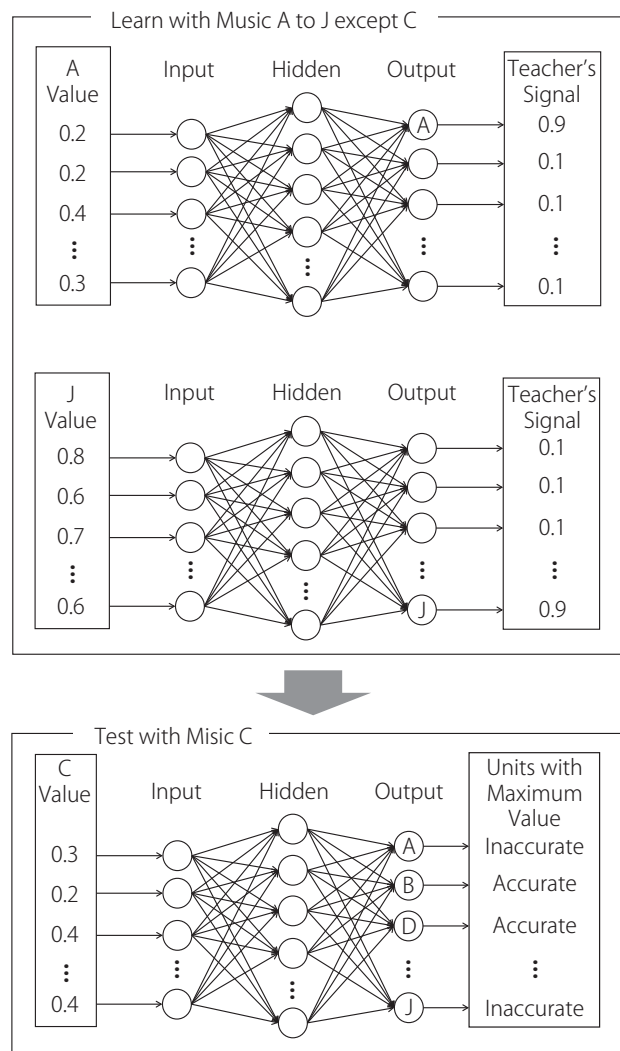


Figure 4: An example of algorithm 2

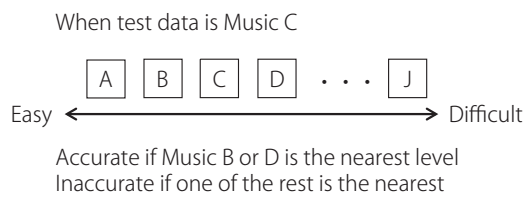


Figure 5: An example of the experiment

its own title and they are arranged according to the difficulty level (Miyagawa, 2003). The titles of the ten music used in the experiment are as follows (in the ascending order of difficulty): *La candeur*, *Arabesque*, *Petite reunion*, *Innocence*, *La gracieuse*, *La chasse*, *La bergeronnette*, *Adieu*, *La Styrienne*, and *La Babillarde*.

### 3.3 Method of the experiment

We employ the nearest neighboring method and the neural network to estimate the difficulty level of a given piece of music nearest to that of music in a given music group arranged according to the known difficulty level. We investigate three sets of attributes: the attributes in the existing studies, the first attribute set and the second attribute set.

#### 3.3.1 Attributes in the existing studies

We investigate the attributes in the existing studies to compare the effectiveness with our proposed attribute sets. We applied following revisions to the former in order to calculate Mahalanobis distance:

- Finger number: the ratio of the notes with finger numbers to all the notes in the score.
- Accidental annotation: the ratio of the notes with accidental annotation to all the notes in the score.
- Tone difference: the mean value and the standard deviation value of the tone difference in the score.
- Change in rhythm: the ratio of the change of notes accompanied with change in rhythm to all the change of notes in the score.
- Dotted note: the ratio of the notes with dot to all the notes in the score.
- Staccato: the ratio of the notes with staccato mark to all the notes in the score.
- Grace note: the ratio of the grace notes to all the notes in the score.
- Continuant: the ratio of the continuants to all the notes in the score.
- Black key: the ratio of the notes played with black key to all the notes in the score.
- Distance: the ratio of the notes with distance to all the notes in the score.
- Rest: the ratio of the rests in the score.

- Chord: the ratio of the notes composing a chord to all the notes in the score.
- Black key without accidental notation: the ratio of the notes played by using black key without accidental notations to all the notes in the score.
- Fingering: the ratio of the tone transition with passing under, crossing over, or the same note played with different fingers to all the tone transitions.
- Nth note: the ratio of the half notes, quarter notes, eighth notes, sixteenth notes, and thirty second notes to all the notes in the score.

#### 3.3.2 Experiment using the nearest neighboring method

In this study, we judge the nearest difficulty level between two pieces of music according to the distance. In the experiment using the nearest neighboring method, we calculate the distance between music used as a test data and the other music pieces in the learning data. The music in the learning data with the nearest distance to the test data is estimated as having the closest difficulty level. If the actual difficulty level of the music is similar to the test data, the result is judged accurate. If not, the result is judged inaccurate.

#### 3.3.3 Experiment using the neural network

In the experiment using the neural network, we input test data into the neural network system which has completed the learning through learning data to find music with the closest difficult level based on the output data. As teachers' signals for teaching the neural network by using the learning data, the value of 0.9 is assigned to the accurate unit and the value of 0.1 is assigned to inaccurate units. Music which the unit with maximum value in the output layer represents is judged as the music with the closest difficulty level to the test data which is input into the system.

The result of the experiment using the neural network can vary according to a default value, so we performed 100 trials under the same condition. The mean value of these results is considered as an accuracy rate. We use optimum parameters obtained through preliminary trials for each attribute set.

### 3.4 The results of the experiments

Table 1 shows the accuracy rates of the three experiments which finds accurately the music with closest difficulty level. In both cases using the nearest neighboring algorithm and the neural network, the accuracy rates are higher for the two

Table 1: Accuracy rate

	Nearest neighboring method	Neural network
Existing attributes	50.0 %	47.2 %
1st attribute set	60.0 %	60.4 %
2nd attribute set	70.0 %	72.4 %

attribute sets we proposed compared to the attribute in the existing studies. Specifically, the combination of the neural network and the second attribute set resulted in 72.4 % accuracy rate, the highest rate in the experiments.

#### 4. Analysis of the results

##### 4.1 Comparison of the accuracy rates

For the reason of the higher accuracy rate of the first attribute set compared to the attributes in the existing studies, it is reasonable to suppose that the former set was able to better reflect the difficulty of the music than the latter by addressing the problems of insufficient information of fingerings and deviation of the composing attributes.

For the reason of the higher accuracy rate of the second attribute set compared to the attributes in the existing studies and the first attribute set, we suppose as follows: while the fingering attributes in the attributes in the existing studies and the first set focus only on the particular movement of fingers, the second set takes into account the ease of finger movement on all the transitions of notes in the score by combining note difference and the fingerings. It is reasonable to suppose, hence, the second attribute set can comprehensively estimate the difficulty of the fingerings.

##### 4.2 Analysis of the failure

In this section, we analyze the music which the system could not accurately estimate the difficult level. For the results of the experiment using the neural network, they could vary according to a default value. So the music to be analyzed are those which were inaccurately estimated 80 times out of 100 trials.

Table 2 shows the list of music which are inaccurately estimated. The difficult level of *Petite reunion* were inaccurately estimated in all the six experiments combining 3 attribute sets and two algorithms. We suppose attributes such as tone

difference of the notes composing the chord and the calculation of the cost were not appropriate enough. When playing this music, the right hand of the player plays the chord most of the time, so the calculation method of the chord has significant influence. Our study used tone difference and the mean cost of the each notes composing the chord. We need to revise the calculation method of the chord.

Secondly, we analyze the reason for the similarity of music inaccurately estimated between the attribute in the existing studies and the first attribute set. The accuracy rates of the experiments using the first attribute set are higher than those of the existing attributes. However, there are four music which were estimated inaccurately in the experiments using both attribute sets. It is reasonable to suppose that the mean value and the standard deviation value of the tone differences employed in both sets wielded significant influence on difficulty level estimation since variations of these values were larger than those of other attributes.

Thirdly, for the four music which were inaccurately estimated their difficulty levels, we analyze three music, *La candeur*, *Adieu*, and *La chasse*. The system inaccurately estimated that these three music were similar in the difficulty level for *Adieu*, *La candeur* and *Babillarde* respectively (Table 3). The reason for the inaccurate combination of *Adieu* and *La candeur* could be the fact that both music are played with constant rhythm without dynamic arm movements. Fingerings are similar. However, *Adieu* has more short notes which requires quick fingerings so the difficult level is higher than that of *La candeur*. For the inaccurate combination of *La chasse* and *Babillarde*, the fingerings are similar but *Babillarde* has more short notes which requires quick fingerings. So the difficult level is higher than that of *La chasse*. For these reasons, in the experiments using the second set, the system often estimates inaccurately the music with similar fingerings as having the closest difficulty level. The second attribute set takes into account two attributes of fingerings and rhythm change, and fingerings attributes account for 87.5 % of the second set. For these two reasons, the system inaccurately estimated the difficulty level of music with similar fingerings. Additionally, the second attribute set does not take into account the length of notes, so the set does not offer information of the length of the notes or tempo of the music as the attribute contributing to the difficulty level. For further research, we need to add another attribute on the length of tone to the second attribute set.

Table 2: Music judged inaccurate

	Nearest neighboring method	Neural network
Existing attributes	<i>La candeur</i>	<i>La candeur</i>
	<i>Arabesque</i>	<i>Arabesque</i>
	<i>Petite reunion</i>	<i>Petite reunion</i>
	<i>Innocence</i>	<i>Innocence</i>
	<i>La gracieuse</i>	<i>La chasse</i>
1st attribute set	<i>La candeur</i>	<i>La candeur</i>
	<i>Arabesque</i>	<i>Arabesque</i>
	<i>Petite reunion</i>	<i>Petite reunion</i>
	<i>La gracieuse</i>	<i>La chasse</i>
2nd attribute set	<i>La candeur</i>	<i>La candeur</i>
	<i>Petite reunion</i>	<i>La chasse</i>
	<i>Adieu</i>	<i>Adieu</i>

Table 3: List of music mistakenly chosen by using second attribute set

Input music	Output music
<i>La candeur</i>	<i>Adieu</i>
<i>Adieu</i>	<i>La candeur</i>
<i>La chasse</i>	<i>Babillarde</i>

## 5. Conclusion

Our study proposed a method of arranging music in a given music set in order to build a system which recommends to a piano self-learner music which fits his/her preference and difficulty level out of a given music group. Unlike one of the existing studies using only multiple regression to derive an evaluation function for difficulty estimation, we employed two algorithms to make better estimation of the difficulty level of music. We performed experiments to verify the possibility of estimating music with similar difficulty level from the music group arranged according to the known difficulty level. The experiments with the combination of the second attribute set and the neural network algorithm resulted in 72.4 % accuracy rate of the difficulty estimation.

Our study used ten music listed in *Burgmüller's 25 studies for piano*. This is not enough. We need to expand the number of music for future experiments. Additionally, by refining the attributes and revising the algorithms, we would like to improve the accuracy rates of the system.

## Note

<sup>(1)</sup> A similar existing study (Kasimi et al., 2007) does not list all the score. Our study has calculated all the cost and made the list under the instruction of a piano instructor.

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## Appendix

Following three tables show the cost of note transition representing fingering easiness for the second attribute set: Table 4 shows the cost of tone transition without passing under or crossing over; Table 5 shows the cost of tone transition with passing under or crossing over and when the first tone is played by a white key; and the Table 6 shows the cost of tone transition with passing under or crossing over and when the first tone is played by a black key.

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Table 4: Transition cost without passing under or crossing over

	Tone difference																
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
T&I	0	0	0	1	1	2	2	3	4	5	5	6	6	6	6	6	6
T&M	1	2	1	0	0	0	0	1	2	3	4	5	6	6	6	6	6
T&R	3	5	5	2	1	1	0	0	1	2	3	4	5	5	5	6	6
T&L	5	5	4	3	2	2	1	0	0	1	2	3	4	5	5	6	6
I&M	3	0	0	1	2	3	4	5	6	6	6	6	6	6	6	6	6
I&R	5	4	2	1	0	1	1	2	4	5	6	6	6	6	6	6	6
I&L	5	5	5	4	2	1	0	1	2	3	4	5	6	6	6	6	6
M&R	5	2	0	2	3	4	5	6	6	6	6	6	6	6	6	6	6
M&L	5	5	2	2	0	1	1	2	3	5	6	6	6	6	6	6	6
R&L	5	5	0	1	2	3	4	5	6	6	6	6	6	6	6	6	6
Others	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Note: T stands for thumb, I for index finger, M for middle finger, R for ring finger and L for little finger.

Table 5: Transition cost with passing under or crossing over and when the first tone is played by a white key

	Tone difference																
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
T&I	n.a.	4	1	4	3	5	4	5	5	6	6	6	6	6	6	6	6
T&M	n.a.	4	1	4	2	5	4	6	5	6	6	6	6	6	6	6	6
T&R	n.a.	4	1	4	3	5	5	6	6	6	6	6	6	6	6	6	6
T&L	n.a.	6	3	6	5	6	6	6	6	6	6	6	6	6	6	6	6
Others	n.a.	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6

Note: T stands for thumb, I for index finger, M for middle finger, R for ring finger and L for little finger.

Table 6: Transition cost with passing under or crossing over and when the first tone is played by a black key

	Tone difference																
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
T&I	n.a.	1	4	2	5	3	5	5	6	6	6	6	6	6	6	6	6
T&M	n.a.	1	4	2	5	3	5	5	6	6	6	6	6	6	6	6	6
T&R	n.a.	1	4	3	6	5	6	6	6	6	6	6	6	6	6	6	6
T&L	n.a.	3	5	4	6	6	6	6	6	6	6	6	6	6	6	6	6
Others	n.a.	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6

Note: T stands for thumb, I for index finger, M for middle finger, R for ring finger and L for little finger.