Simulation of auto valet parking using automatic pallets

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Abstract

Multi-story car parking systems were invented to increase the capacity of car park buildings. However, it is pointed out that the systems have problems such as accidents (in-park movement, theft), searching for parking spaces, boarding/deboarding at narrow designated places, and difficulties in using for beginners and elderly people. To solve these issues, the authors of this paper propose to introduce electric pallets that are called automatic pallets for transporting vehicles and realize automated valet parking that automates procedures for entrance and exit. With the introduction of automatic pallets, a wide variety of vehicles can fit into specific configurations. This facilitates automation. Also, the authors propose a system for improving the time efficiency of exit operations. The time efficiency of automatic pallet exiting is improved by automatically repeating movement to approach the exit side elevator after entering. However, it is unclear how good the time efficient exit operation in that system is. Also, the authors decided to simulate the proposed auto valet parking to find out what kind of problems there are. A simulation was performed on the entering reception and the exit operation. In addition, although random numbers are used in the simulation, two random number generation methods were used for comparison.

Key words

simulation of auto valet parking, time efficient exit operation, parking location determination method, automatic pallet, automated driving

1. Introduction

Today, land prices are high in business districts and downtown areas, making it difficult to construct profitable parking lots. In such places, from the viewpoint of profitability, a multi-story parking lot with a larger capacity is often built. The authors proposed a multi-story car park to maximize utilization efficiency in three dimensional space (Funase et al., 2022a). However, expansion of the scale of multi-story parking lots could worsen user experiences. For example, problems such as accidents (in-park movement, theft), searching for parking spaces, boarding/deboarding at narrow designated places, and difficulties in use for beginners and elderly people arise. To solve such problems, the authors proposed auto valet parking using automatic pallets (Funase et al., 2022b). And, the authors proposed a parking location determination method to realize high time efficient exit operations (Funase et al., 2022c). In addition, an automatic pallet movement method that improves exit time efficiency is proposed (Funase et al., 2022d). However, it is unclear how good the time efficient exit operation is. Also, it was decided to simulate the proposed auto valet parking to find out what kind of problems there are. Previous studies (Saito, 2018; Autonomous Driving Lab., 2020; Funase et al., 2021b; 2021c; 2021d) have pointed out that the accuracy of research on automated driving (Ministry of Land, Infrastructure, Transport and Tourism, 2019; Tanikawa, 2019; Yamazaki et al., 2019; Kitagawa Corporation, 2021; Japan Automobile Research Institute, 2021) is still insufficient. Reasons for low accuracy include the fact that the vehicles that use the parking lot are diverse and differ in performance and appearance, and that entrance/exit procedures by automated driving based on instructions from mobile phones have not yet reached the stage of practical use.

To improve accuracy, Funase et al. made a proposal to realize an automated valet parking system with high-precision automated driving by limiting the target of automated driving to automatic pallets for transporting vehicles. Based on the system (Funase et al., 2022b) that allows cars to be parked to be placed on an automatic pallet to enter and exit the parking lot, we first made improvements even easier to make the movement of the automatic pallet to the left/right and the front/back. This has made it possible to increase the number of parking spaces per floor. In the entrance procedure, after the user gets out of the car in the drop-off area and completes the driver's facial recognition registration, the car is loaded on to an automatic pallet and moved to the parking location determined by the automated valet parking system. For the user, the entry procedure is completed simply by getting out of the car and registering the face, if there is no queue, it is such a short time that it is barely noticeable. Even if it takes an enormous amount of time for the automatic pallet to move to the parking location in the multi-story car park, the user does not have to wait because the operation is automatic. The time efficiency of automated valet parking, which is a problem for users, is the waiting time required from the time a user orders his/her car to exit until the vehicle is actually delivered. To confirm these times, we simulate some models below.

2. Multi-story car parking

The parking has multiple floors from the 1st basement floor to the m^{th} floor and uses automatic pallets on which cars are parked. The surfaces of each floor are treated so that pallet wheels do not slip. Two elevators are installed for pallets transport, each connected to the floor entrance and exit. These elevators are for the exclusive use of entrance and exit respectively. And, these elevators are waiting on the 1st basement floor when not in use. All automatic pallets are assigned with unique serial numbers from 1 with which entrance and exit procedures are managed. A unit space is a space slightly larger than an automatic pallet and can accommodate one automatic pallet. Each unit space is also given a unique serial number. Numbers are assigned in order from the far left end to right end of the 1st basement floor. Then numbers are gradually assigned to the elevator side. When the numbering of the 1st basement floor is completed in this way, the numbering of the 1st floor is performed in the same way, and when that is completed, the numbering of the 2nd and 3rd floors is repeated. Each floor is divided into these unit spaces. A unit space that the automatic pallet can access directly from the passage is called a basic unit space. A unit space whose four sides do not face the passage is called a complex unit space (*No.k*). When automatic pallets are parked in all basic unit spaces adjacent to the complex unit space (*No.k*), the automatic pallet in the basic unit space that is obstructing the movement of the complex unit space is moved first. By doing so, a passage from *No.k* unit space to the exit is created. Furthermore, if the automatic pallet parked in the *No.k* unit space goes out to the exit, the automatic pallet that was moving earlier is returned. The automatic pallet that was moved first needs to be moved for a total of 4 frames (Figure 2). In other words, the movement of *No.k* in this case is added with the movement of four frames. Also, the unit spaces that can be parked are only basic unit spaces and complex unit spaces, and parking in the passage is not permitted. Each floor has a passage connecting the entrance and exit elevators. Passage unit spaces have markers pointing to the exit embedded and cannot accept automatic pallets with cars. Ideally, the passage should be a tree structure with the exit elevator as the root node. This is because creating a closed-circuit passage would reduce the parking space available for the automatic pallet. A passage with a tree structure has only one route from any unit space on the passage to the exit, so the distance to the exit is the shortest. Although there are dead ends in the branch passages, the passages are set so as to minimize the complicated unit space. The passages on each floor are located in the same place. In other words, the

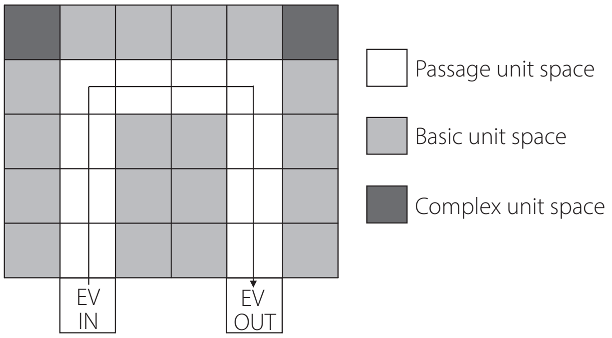
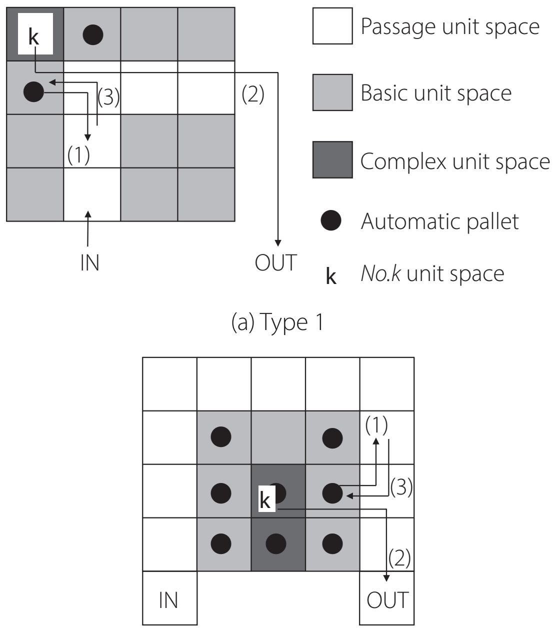


Figure1: Multi-story car parking

Note: 1st basement floor and mth floors above ground, 7 × 6 unit spaces per floor.

structure of each floor in two-dimensional space is exactly the same. The position of each unit space is tabulated so that three-dimensional coordinate values can be obtained from the unit space number. Specifically, if the unit space number is *t*, it is expressed in three dimensions (*Cx(t)*, *Cy(t)*, *Cz(t)*). Here, the x-coordinate *Cx(t)* represents the row number, the y-coordinate *Cy(t)* represents the column number, and the z-coordinate *Cz(t)* represents the floor.

The number of unit spaces on the shortest passage from *No.j* basic unit space to the exit elevator (including the space of the elevator as (1) is the distance to the exit for *No.j* basic unit space. It is called the exit distance *E(j)* of the basic unit space of *No.j*. The exit distance *E(k)* of the *No.k* complex unit space is a value obtained by adding 1 to the shortest distance from the adjacent basic unit space to the exit {*No.j1*, *No.j2*, · · ·, *No.je*}. However, in the case of the basic unit space where the



(a) Type 1

(b) Type 2

Figure 2: Procedure for creating a passage from a complex unit space (*No.k*) to the exit

automatic pallet is parked, add 4 to the shortest distance to the exit. This uses the operation of Figure 2. Four is added because it takes two moves to evacuate and two moves to get back.

Funase et al. (2022d) showed an automatic pallet movement method that improves the time efficiency of auto valet parking. This is shown in the procedure for creating a new passage from the basic unit space (*No.k*) to the exit in Figure 3.

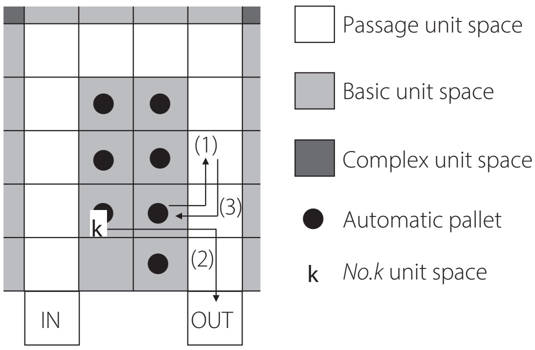


Figure 3: Procedure for creating a new passage from the basic unit space (*No.k*) to the exit

3. Automated valet parking for comparison

Here, an outline of three automated valet parking systems to be compared will be described. All of them use automatic pallets. For details, refer to Funase et al. (2022b; 2022c; 2022d) respectively.

- Type 1 automated valet parking:
  This type is automated valet parking using an automatic pallet proposed in the paper (Funase et al., 2022b). The location of the parking lot is determined by random numbers. Do not move to improve exit time efficiency.
- Type 2 automated valet parking:
  This type is automated valet parking using an automatic pallet proposed in the paper (Funase et al., 2022c). For the initial parking location, select the basic unit space closest to the exit on the 1st basement floor from among the empty unit spaces. In addition, automatic pallets far from the exit are sequentially moved to the exit side to the unit space emptied by the exit operation (implemented according to the exit distance table). For the exit operation, go through the aisle to the exit.
- Type 3 automated valet parking:
  This type is automated valet parking using an automatic pallet proposed in the paper (Funase et al., 2022d). This automated valet parking proposes an automatic pallet movement method that improves the exit time efficiency by adding a new passage from the basic unit space (*No.k*) to the exit as shown in Figure 3 in addition to Type 2 automated valet parking.

4. Common simulation conditions for each automated valet parking

(1) All three types of automated valet parking have a Poisson distribution (*M*) of arrival distribution of vehicles to the multi-story parking lot, and there is only one entrance for the parking reception counter. In addition, it is assumed that the warehousing acceptance time follows an exponential distribution (*M*). Furthermore, since there is no limit to the length of the queue, the Kendall, D. G. symbolic representation of the queue problem up to receipt of goods is *M/M/1(∞)*. However, the order of entrance and exit is not FIFO (First-In First-Out). Cars are entered in the order of acceptance of the entry and exited in the order of the exit request. Next, let us consider the queue problem from the end of reception of car entry to the end of exit of the car. The service time is the parking time from the end of reception of entry to the end of exit and is assumed to follow a Poisson distribution. However, the service time includes the time required for exit. The number of counters are the number of unit spaces (*S*) that can be parked in the multi-story car park, and only basic unit spaces are used this time. Also, there is no limit on the length of the queue. Therefore, the symbol notation is *M/M/S(∞)*.

(2) In events that occur randomly in the flow of time, if the probability of one event occurring within a small time interval is extremely small and the occurrence of each event is unrelated to other factors, the time intervals at which such events occur follow an exponential distribution. Many random events that occur in relation to time in everyday life satisfy this assumption and follow an exponential distribution. For example, it is known that time intervals such as the arrival of an order at a store, the arrival of a customer, the arrival of a patient at a hospital, the occurrence of a traffic accident, etc. all follow an exponential distribution (Kishida, 1974).

The following below explains how to generate exponentially distributed random numbers. First, the probability density function $f(x)$ of the exponential distribution is

$$f(x) = ae^{-ax} \quad (a > 0, x \geq 0) \tag{1}$$

Its cumulative distribution function $F(x)$ is

$$F(x) = \int_0^x f(x)dx = 1 - e^{-ax} \tag{2}$$

Also, average $E(x)$ is the following

$$E(x) = \int_0^\infty xf(x)dx = \int_0^\infty axe^{-ax}\,dx = W \tag{3}$$

Here, by using the integration by parts method,

$$W = 0 + \int_0^\infty e^{-ax}\,dx = 1/a \tag{4}$$

Here, from $0 \le F(x) \le 1$ to $0 \le 1 - F(x) \le 1$, the uniformly distributed random number (*RND(1)*) in the interval (0, 1) is equal to $1 - F(x)$ far. As a result is

$$RND(1) = 1 - F(x) = 1 - (1-e^{-\alpha x}) = e^{-\alpha x} \quad (5)$$

formula becomes simpler. Therefore,

$$logRND(1) = -\alpha x$$
$$x = (-1/\alpha) \cdot logRND(1) \quad (6)$$
$$x = -E(x) \cdot logRND(1)$$

If *E(X)* is the average parking time per car, this will be the parking time of the car derived by random numbers.

(3) For uniformly distributed random numbers in the interval (0, 1), the following two types of extended rand() and Xorshift were used in Experiments 1 to 3 (Kobayashi, 2022), and from Experiment 4 onwards, the extended rand () is used.

- Uniform random number by extended rand():
  By using the built-in function rand() of the C language and dividing rand()+0.5 by RAND_MAX+1, it is evenly distributed without being biased toward both ends. However, since the value of RAND_MAX is relatively small depending on the processing system, a uniform random number is calculated using multiple values of the rand() function as follows.
  ```
  double rand(){
  double m, a;
  m = RAND_MAX + 1;
  a = (rand() + 0.5) / m;
  a = (rand() + a) / m;
   return (rand() + a) / m;
  }
  ```
- Uniform random numbers by Xorshift:
  Xorshift is a pseudo-random number generation algorithm proposed by George Marsaglia in 2003, and can quickly calculate pseudo-random numbers with good properties only by bit manipulation of integer type variables. Xorshift also has sufficient performance to be used for the Monte Carlo method. However, the quality is slightly inferior compared to the Mersenne Twister.

(4) Events that follow a Poisson distribution are many. Examples are the number of orders per unit time at a store and the number of patients visiting a hospital. It is known that when the arrival distribution follows a Poisson distribution, the distribution of successive arrival intervals follows an exponential distribution (Kimura, 1989). Namely, a Poisson distribution with an average of α arrivals per unit time follows an exponential distribution with an average arrival interval of $1/\alpha$. And, if α is the average number of arrivals per unit time, the value of equation (6) is the time interval between vehicle arrivals.

(5) Assume that the speed of the automatic pallet is constant, and the moving time is proportional to the number of moving frames of unit spaces moved. Also, assume that the elevator moving time is proportional to the number of floors moved, and for convenience the moving time of the automatic pallet per unit is the same as the elevator moving time per floor. In short, it is a time per a moving. However, every time the elevator is used, a constant γ time (γ times the single time) is added as preparation time including the time to call the elevator, the waiting time, the time to get on and off, etc. In addition, when the entrance elevator is not in operation, it waits on the 1st basement floor. When receiving an instruction to entrance operation, the elevator loads the automatic pallet, moves to the target floor, unloads the pallet, and then automatically returns to the 1st basement floor. In addition, when the exit elevator is not in operation, it waits on the 1st basement floor, and when it is instructed to exit operation, it moves to the target floor, loads the automatic pallet, and then automatically returns to the 1st basement floor. In other words, if the target floor is the t floor, the elevator moves 2t floors. The γ time is added to it.

5. Simulation experiment of auto valet parking

In the parking reception (entrance reception) processing, a vehicle that has arrived at the entrance is parked at the position of the turntable where the automatic pallet is embedded. Then, the user gets out of the car, locks it, and registers the driver's face with the authentication camera. At that time, the registered face image is associated with information such as date and time, assigned automatic pallet number, etc., and paper on which this information is printed is handed over to the user. This is for the user's memo and is not required at the exit. It is used as information when trouble occurs. And, the face image, the automatic pallet number, and the unit space number of the parking location are associated with each other in the database. After the pallet has been removed, a new automatic pallet is carried and set on the turntable. A series of these operations are performed automatically.

5.1 Experiment 1

Consider the following queuing problem from the arrival of a car in a multi-story parking lot to the end of entry reception of the *M/M/1(∞)* model (Figure 4). There is only one entrance for reception of car entry, which is the counter of this multi-story parking lot. Car arrival follows a Poisson distribution with the number of averages of α per unit time (1 minute). The reception time of car entry follows an exponential distribution with an average of β minutes per car. In addition, there is

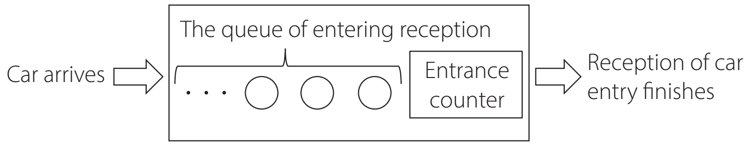


Figure 4: Queuing model at counter of car entry in Experiment 1

no limit on the length of queue for reception of car entry. At this time, the simulation is continued until reception of a cars entry is completed. Next, calculate average of waiting time of the arrival of a car and probability of not waiting. The system is from waiting for reception of car entry to reception of car processing (Figure 4).

5.1.1 Result

From Chapter 4 (4), when arrival distribution follows Poisson distribution, distribution of successive arrival intervals follows an exponential distribution, so the arrival time intervals in this case follow an exponential distribution with an average of *1/α* minutes. What is important in this simulation is not the arrival time interval *BT* of a car, but the *BWT* obtained by subtracting the waiting time *WT* for reception of car entry from *BT*. Figures 5, 6, and 7 show this in terms of time of arrival car, waiting time for reception of car entry, and availability of counter. In other words, from the magnitude relationship between reception time of car entry β and *BWT*, we can see whether the next car will have to wait, whether there will be an available at the counter, and so on.

Also, since the simulation can be started at any time, for the sake of convenience, it is assumed that the first user arrives at the same time as the simulation starts.

The simulation of Experiment 1 can also be obtained by an analytical method (Kimura and Oyabu, 1989). Specifically, it is as follows.

First, traffic density γ is

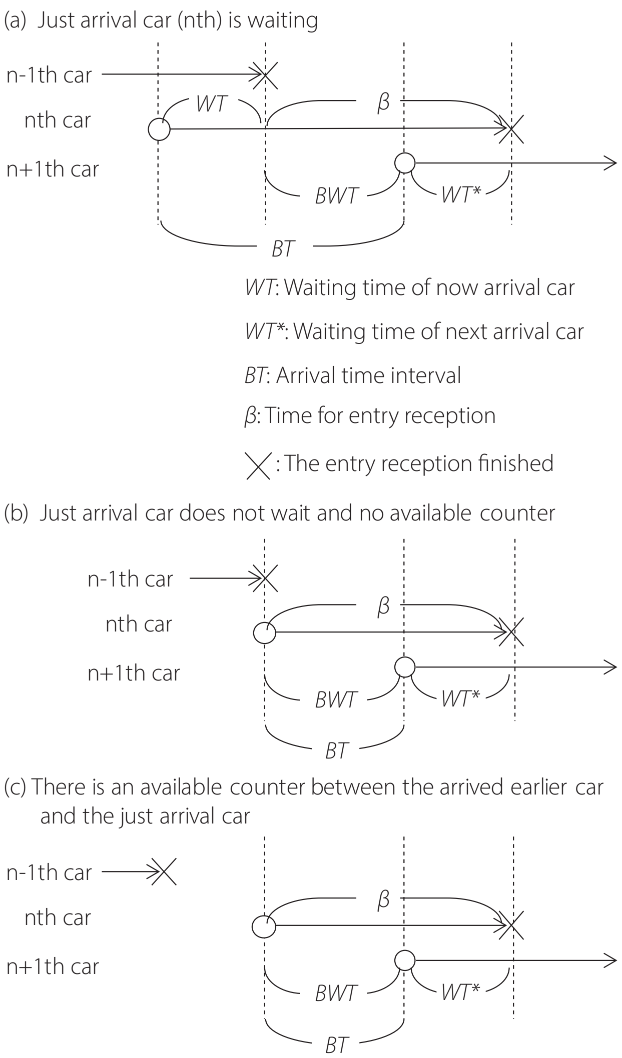


Figure 5: In the case that the next arrival car (order of *n + 1*) has to wait and there is no available counter of car entry between time interval of now arrival car to next arrival car ($\beta > BWT$)

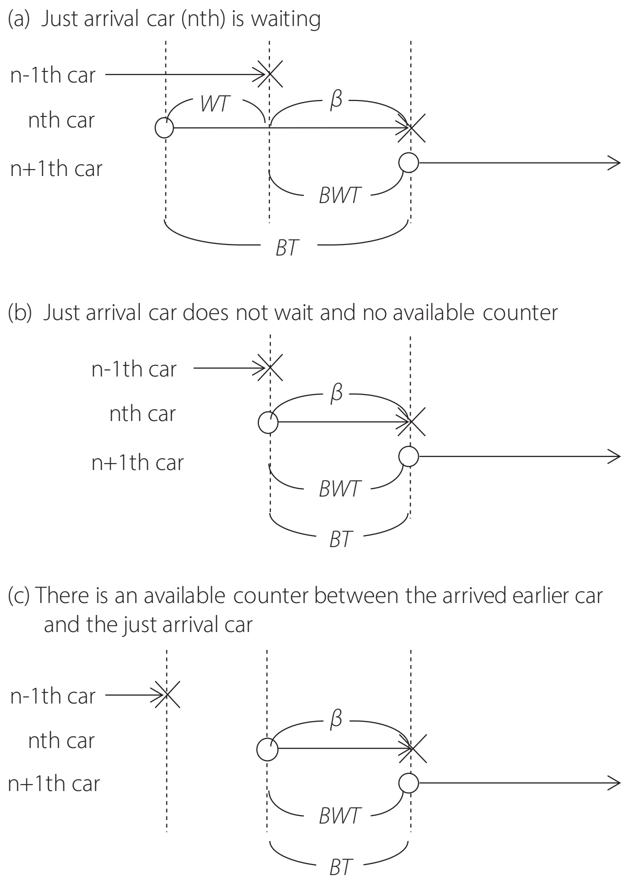


Figure 6: In the case that the next arrival car (order of *n + 1*) has no wait and there is no available counter of car entry between time interval of just arrival car to next arrival car ($\beta = BWT$)

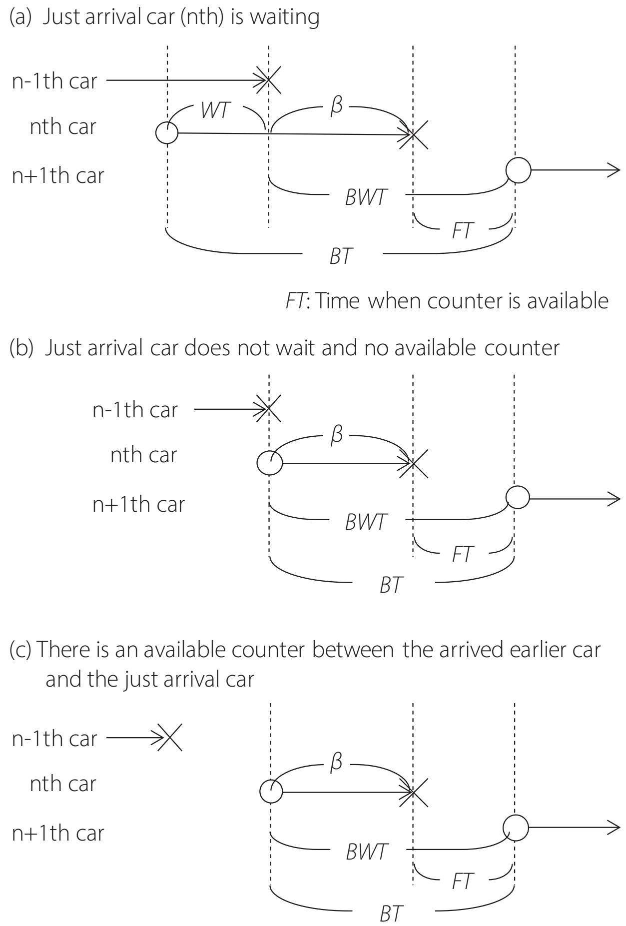


Figure 7: In the case that the next arrival car (order of $n + 1$) has no wait and there is available counter of car entry between time interval of just arrival car to next arrival car ($\beta < BWT$)

$$\gamma = \alpha\beta \tag{7}$$

Probability P_n of n cars in the system is

$$P_n = (\alpha\beta)^n(1 - \alpha\beta) = \gamma^n(1 - \gamma) \tag{8}$$

However, $\gamma < 1, n \geqq 0$
Average number of car $E(n)$ in the system is

$$\begin{aligned} E(n) &= \textstyle\sum_{n=0}^{\infty} nP_n = \sum_{n=0}^{\infty} n\gamma^n(1 - \gamma) \\ &= \gamma(1 - \gamma)(1 + 2\gamma + 3\gamma^2 + 4\gamma^3 + \cdots) \end{aligned} \tag{9}$$

Now assume that $T = 1 + 2\gamma + 3\gamma^2 + 4\gamma^3 + \cdots$

$$\begin{aligned} \gamma T &= \gamma + 2\gamma^2 + 3\gamma^3 + 4\gamma^4 + \cdots \\ T - \gamma T &= 1 + \gamma + \gamma^2 + \gamma^3 + \gamma^4 + \cdots \\ &= (1 - \gamma^\infty) / (1 - \gamma) \\ T(1 - \gamma) &= 1 / (1 - \gamma) \\ T &= 1 / (1 - \gamma)^2 \end{aligned} \tag{10}$$

Therefore, $E(n) = \gamma(1 - \gamma) / (1 - \gamma)^2$

$$= \gamma / (1 - \gamma) \tag{11}$$

Average length of waiting queue (number of cars) $E(m)$ does not include a car which is making a reception and the length of queue is 0 when there is no car in the system or there is only one car.

Therefore,

$$\begin{aligned} E(m) &= \textstyle\sum_{n=1}^{\infty} (n - 1)P_n = \sum_{n=1}^{\infty} nP_n - \sum_{n=1}^{\infty} P_n \\ &= E(n) - (1 - P_0) \\ &= \gamma / (1 - \gamma) - (1 - \gamma^0 (1 - \gamma)) \\ &= \gamma / (1 - \gamma) - \gamma \\ &= \gamma^2 / (1 - \gamma) \end{aligned} \tag{12}$$

Because the average length of the queue in the system is $E(m)$, on average, when the number of $E(m)$ cars arrive, just arrival car completes the entry reception or making reception in the entry counter. Therefore, the average waiting time $E(w)$ for next arrival car in the car entry counter can be obtained by multiplying $E(m)$ by the average arrival time interval $1/\alpha$.

Therefore,

$$E(w) = \alpha / ((1 / \beta)((1 / \beta) - \alpha)) \tag{13}$$

Also, since the probability that there is no wait is P_0.

$$P_0 = 1 - \alpha\beta \tag{14}$$

Figure 8 shows a flow chart for solving the problem of this experiment. Tables 1 and 2 show the run results of the program created from the flow chart of Figure 8. Table 1 shows the results of using extended rand() for uniformly distributed random numbers, and Table 2 shows the results of using Xorshift for uniformly distributed random numbers. The values in these tables are average values of 100 iterations. At first, we used the C built-in function rand(), but the error was too large, so we tried extended rand() and Xorshift. Although the accuracy of the results was improved, the difference between the two was hardly observed. In Table 1, Table 2, Table 3, and Table 4 ● is marked to whichever is superior among the extended rand() and Xorshift uniform distribution random number generation methods. Normally, the Mersenne Twister is used as a replacement for the rand() function, but the above two ways were chosen because it is somewhat difficult to set up and operate. In this paper, one counter of car entry is used because the parking reception process can be completed in a short period of time. However, when $\alpha \cdot \beta > 1$, it is necessary to increase the number of windows.

5.2 Experiment 2

Next, we will consider the queuing problem $M/M/S(\infty)$ from the end of reception of car entry to the end of car exit (Figure 9).

In order to adapt the experiment to the situation of present

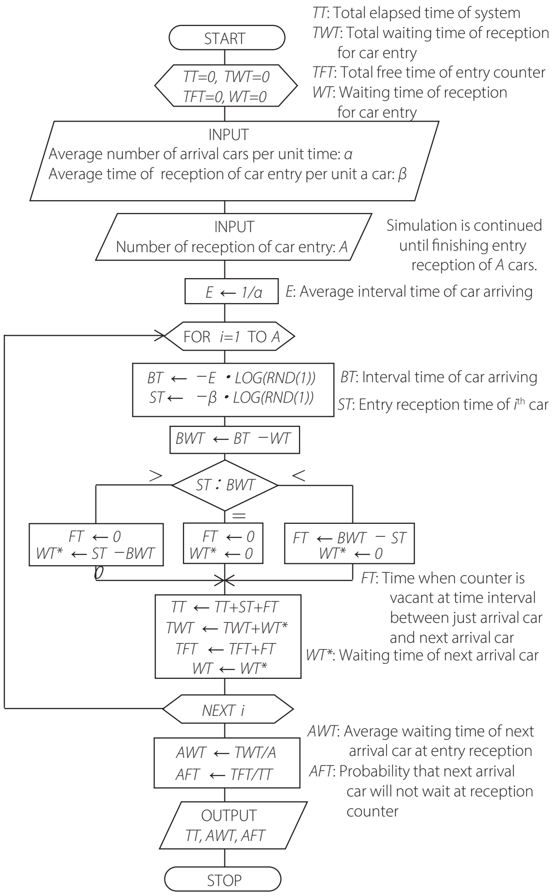


Figure 8: Flow chart for solving problem of Experiment 1

multi-story car parking, only basic unit space is used for parking location, and complex unit space is not used. Therefore, the number of target counters in the multi-story car parking is *S* pieces that is the number of basic unit space. The number of cars which completed entry reception follows Poisson distribution of average *1/E* cars per unit time (1 minute). And, the parking time follows exponential distribution of average *β* minute. However, the parking time includes the time that a car moves from the waiting space (Figure 9) at entry reception to basic unit space and the time until the car arrives at the boarding area at exit operation. Also, there is no limit to the length of queues for parking. At this time, we calculate the average waiting time in the parking queue (see Figure 9) and the average parking time (average storing time) until a car arrives at the boarding area at exit operation which includes waiting time. However, this system (Figure 9) shows from the car waiting for parking in the waiting room to car parking at the counter.

In this experiment, first *S* cars are assigned in order to the counter of basic unit space from Counter 1 to Counter *S*. From the *S* + *1*st car, it will be assigned to the counter that is first available. When there is no available counter, the car will wait at the waiting room. And, simulate until *M * (> S)* cars are parked.

5.2.1 Result

Overall program structure is the same as the case of *M/M/1(∞)* model. First of all, first *S* cars carry out reception of car entry. We calculate the staying time and available time at each counter. Next, find the number and time of the counter that is available earliest. And, calculate the waiting time and available time from the time of car arriving. Figure 10 shows a flow chart of these calculations.

Table 1: *AWT* and *AFT* when *α = 0.2, β = 1* (using extended rand())

A	Simulation value of *AWT*	Theoretical value of *AWT*	Simulation value of *AFT*	Theoretical value of AFT
100	0.240002 (96.0008 %)	0.250000	0.7977997● (99.72496 %)	0.8000000
1000	0.255178 (97.9288 %)	0.250000	0.7991470 (99.89337 %)	0.8000000
10000	0.250271● (99.8916 %)	0.250000	0.7998527● (99.98158 %)	0.8000000

Notes: *A* = Number of cars (Number of running simulation of entry reception); *AWT* = Average waiting time of next arrival car at entry counter. *AFT* = Probability of no waiting; ( ) = Accuracy.

Table 2: *AWT* and *AFT* when *α = 0.2, β = 1* (using Xorshift)

A	Simulation value of AWT	Theoretical value of AWT	Simulation value of AFT	Theoretical value of AFT
100	0.240711● (96.2844 %)	0.250000	0.7963637 (99.54546 %)	0.8000000
1000	0.250832● (99.6672 %)	0.250000	0.8001009● (99.98738 %)	0.8000000
10000	0.250798 (99.6808 %)	0.250000	0.7995312 (99.9414 %)	0.8000000

Notes: *A* = Number of cars (Number of running simulation of entry reception); *AWT* = Average waiting time of next arrival car at entry counter; *AFT* = Probability of no waiting; ( ) = Accuracy.

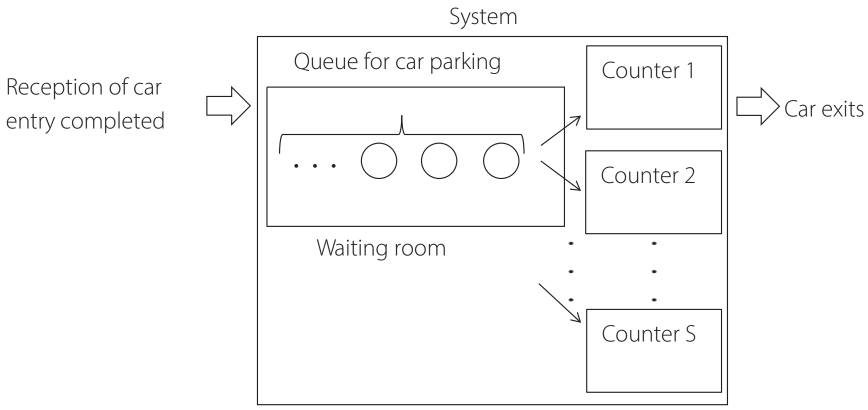


Figure 9: Queue model with some counters of Experiment 2

The simulation of Experiment 2 can also be calculated by an analysis method. Specifically, from the literature of Kimura and Oyabu (1989), the theoretical value *E(w*)* of the average waiting time in the parking queue (average waiting time) and the theoretical value *E(w*)* of the average staying time including the waiting time in the parking queue *E(v)* is as follows:

$$E(w^*) = \beta \cdot r^S \cdot P_0^* / ((S-1)!\,(S-r)2) \qquad (15)$$

$$E(v) = E(w^*) + \beta \qquad (16)$$

Here, $r = \beta / E$

$$P_0^* = 1 / ((\textstyle\sum_{n=0}^{S-1} r^n / n!) + r^s / ((S-1)!\,(S-r))) \qquad (17)$$

Therefore, $r / S < 1$

Table 3 and Table 4 show running results of the program that is created from the flowchart of Figure 10. Table 3 shows results that used extended rand() for uniformly distributed random numbers, Table 4 shows results that used Xorshift for uniformly distributed random numbers. The values in these tables are average values of 100 iterations.

Experiments 1 and 2 give an overview of the queuing problems in parking lots. It is assumed that there is one entering reception desk and that the number of cars arriving per unit time is α, and follows the Poisson distribution. Also, assuming that the entering reception time per car follows the exponential distribution of *β*, we calculated the average waiting time for the next arriving car and the probability that it will not be waiting. In this case, the server counter is a basic unit space, the number of the counter is *S*, the average arrival time interval of automatic pallets entering the parking lot

Table 3: *AW* and *ATS* when *S = 2, E = 3, β = 4* (using extended rand())

*M**	Simulation value of AW	Theoretical value of AW	Simulation value of ATS	Theoreticalvalue of ATS
100	2.960047● (92.50146 %)	3.200000	6.989210● (97.07236 %)	7.200000
1000	3.086164 (96.44262 %)	3.200000	7.072541 (98.22973 %)	7.200000
10000	3.221516● (99.32762 %)	3.200000	7.221829● (99.69681 %)	7.200000

Notes: *M** = Number of car simulating; *AW* = Average waiting time (minute) of car in parking queues; *ATS* = Average time of stay (minute) including waiting time in parking queues; ( ) = Accuracy.

Table 4: *AW* and *ATS* when *S = 2, E = 3, β = 4* (using Xorshift)

*M**	Simulation value of AW	Theoretical value of AW	Simulation value of ATS	Theoreticalvalue of ATS
100	2.820554 (88.14231 %)	3.200000	6.800954 (94.45769 %)	7.200000
1000	3.231280● (99.02250 %)	3.200000	7.259480● (99.17388 %)	7.200000
10000	3.238991 (98.78153 %)	3.200000	7.248392 (99.32788 %)	7.200000

Notes: *M** = Number of car simulating; *AW* = Average waiting time (minute) of car in parking queues; *ATS* = Average time of stay (minute) including waiting time in parking queues; ( ) = Accuracy.

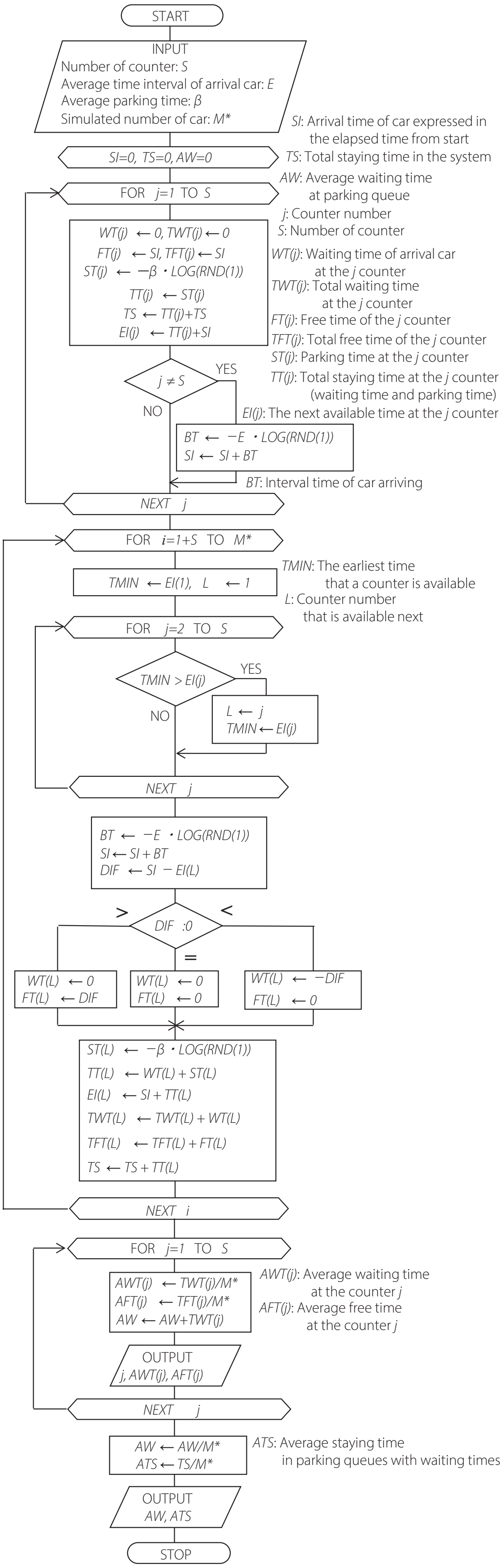


Figure 10: Flow chart for calculating results of Experiment 2

after the entry reception is completed following the Poisson distribution of *E*, and if the average parking time is an exponential distribution of *β*, we calculated the average waiting time for parking and the average staying time including waiting time. Both simulations were confirmed not only by the random number generation method but also by comparing the results from the analysis method.

Next, we deal with the exit operation time of a car, which is the most important concern for the user. This is the time from when the user goes to the boarding area and requests to exit the car until the car arrives. For Experiment 3 and subsequent experiments, consider the case where *M** cars enter the parking lot in order, and then all the cars exit the parking lot in order using uniform random numbers. In the following, the exit time of each type of automated valet parking is determined experimentally and compared. However, the boarding area is assumed to be near the exit on the 1st basement floor, and the exit time is the movement time to the exit on the 1st basement floor.

5.3 Experiment 3

The target multi-story car park has 1st basement floor and m above-ground floors as shown in Figure 1. However, the unit space used as a parking space will be only the basic unit space. Figure 11 shows the distance *E(j)* from the basic unit space of *No.j* on the 1st basement floor to the exit on the same floor. The automated valet parking used is Type 1. At this time, the total distance of exit operation the parking lot of *M** cars is calculated.

5.3.1 Result

Assuming that the number of unit spaces per floor is *U*, the unit space *No.* on the *k* floor directly above the basic unit space *No.j* on the 1st basement floor is $U \cdot k + j$, and the exit distance is the same. That is $E(U \cdot k + j) = E(j)$ for $k = 1, 2, 3, \ldots, m$. Figure 12 shows a flowchart that calculates the total distance of exit operation. Also, Table 5 and Table 6 show the running results of a program created from the flowchart in Figure 12. Table 5 shows the results of using extended rand() for uniformly distributed random numbers, and Table 6 shows the results of using Xorshift for uniformly distributed random numbers.

5.4 Experiment 4

Using Type 2 auto valet parking and others are the same as Experiment 3. At this time, calculate total distance of exit operation. However, automatic pallet movement during parking location improvement will not interfere with movement during exit. In addition, the automatic pallets at the time of improvement will operate in parallel.

(1) -2	(2) 10	(3) 9	(4) 8	(5) 7	(6) -2
(7) 10	(8) -1	(9) -1	(10) -1	(11) -1	(12) 7
(13) 11	(14) -1	(15) 9	(16) 6	(17) -1	(18) 6
(19) 12	(20) -1	(21) 12	(22) 5	(23) -1	(24) 5
(25) 13	(26) -1	(27) 13	(28) 4	(29) -1	(30) 4
(31) 14	(32) -1	(33) 14	(34) 3	(35) -1	(36) 3
(37) 15	(38) -1	(39) 15	(40) 2	(41) -1	(42) 2
	EV IN			EV OUT	

(j) *E(j)*	*j*: No. *E(j)*: Exit distance

The No.*j* of unit space is a passage,
when the term *E(j)* is −1.
The No.*j* of unit space is a complex unit space,
when the term *E(j)* is −2.

Figure 11: *No.* of unit space on 1st basement floor and exit distance of basic unit space in Figure 1

5.4.1 Result

Figure 13 shows a flowchart that calculates the total distance of exit operation. Also, Tables 7 and 8 show the run results of the program created from this flowchart.

5.5 Experiment 5

Using Type 3 auto valet parking and the difference from Type 2 is that the exit distance in Figure 11 becomes the exit distance in Figure 14. At this time, calculate the total distance of exit operation.

5.5.1 Result

Used flowchart is the same as the flowchart of Figure 13 and the data used is the exit distance in Figure 14. Table 9 and Table10 shows this run result.

Figure 15 shows a comparative diagram plotting the data in Tables 5, 7 and 9 when the extended rand() is used. And, Figure 16 shows a comparative diagram plotting the data in Tables 6, 8 and 10 when Xorshift is used.

From the results of Experiments 3, 4, and 5, it can be seen that the proposed automatic valet parking Type 3 has the shortest exit distance.

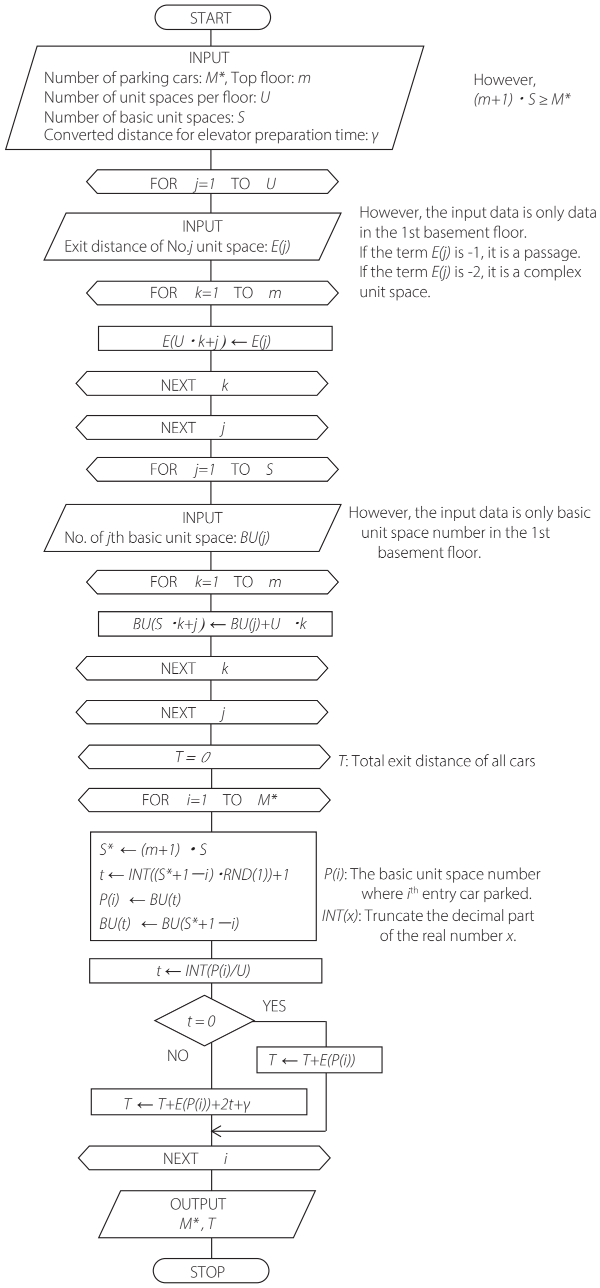


Figure 12: Flowchart that calculates results of Experiment 3

6. Conclusion

Analytic methods are useful when solving a queuing problems. However, an analytical method cannot be used when making changes that do not fit formula. Simulation is more flexible in such cases. In this paper, Experiment 1 and Experiment 2 were simulated to receive an overview of the queuing problem in the parking lot. Both Experiment 1 and Experiment 2 were simulated highly accuratly, and both experiments gave highly accurate results. In particular, Experiment 1 yielded accurate values with about 100 iterations. Also, we used extended rand() and Xorshift for generating uniform random numbers, but there was no difference in accuracy. For the *AFT* simulation values in Table 2, 1000 iterations were better than 10000 iterations. Also, even for the *AW* simula-

Table 5: *T* when *m = 4, U = 42, S = 26, γ = 1* (Experiment 3 of using extended rand())

*M**	Total of exit distance for all parking cars*(T)*
10	159
20	260
30	379
40	499
50	637
60	778
70	930
80	1066
90	1180
100	1318
110	1455
120	1598
130	1730

Notes: *M** = Number of parking cars; Using extended rand() in uniform random number.

Table 6: *T* when *m = 4, U = 42, S = 26, γ = 1* (Experiment 3 of using Xorshift)

M*	Total of exit distance for all parking cars*(T)*
10	107
20	232
30	364
40	493
50	646
60	779
70	946
80	1084
90	1212
100	1322
110	1481
120	1598
130	1730

Notes: *M**= Number of parking cars; Using Xorshift in uniform random number.

tion values in Table 4, 1000 iterations were better than 10000 iterations. These phenomena can be attributed to the already converged values and errors. The simulation results in Experiments 1 and 2 are useful for investigating the situation of the proposed automatic valet parking. In Tables 5 and 6, *T* is 1730 when *M** (the number of parking spaces) is 130, which is the maximum parking capacity. It is the total moving time of all cars moving from all basic unit spaces to exit on the 1st basement floor. In other words, in Type 1 automated valet parking,

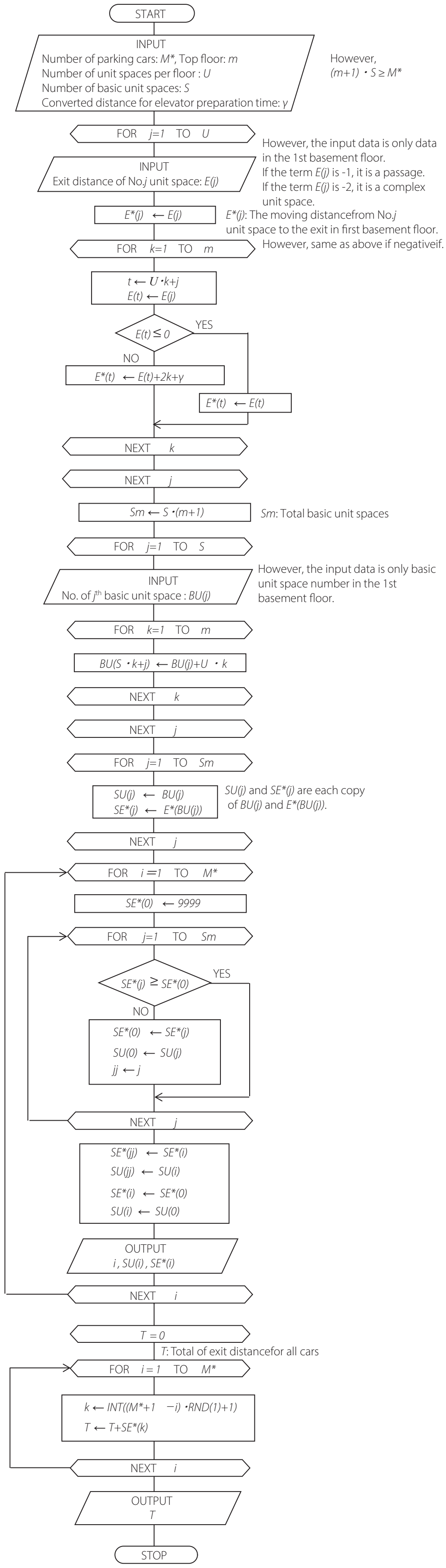


Figure13: Flowchart that calculates results of Experiment 4

Table 7: *T* when *m = 4, U = 42, S = 26, γ = 1* (Experiment 4 of using extended rand())

*M**	Total of exit distance for all parking cars*(T)*
10	37
20	88
30	149
40	214
50	324
60	431
70	518
80	667
90	742
100	906
110	1018
120	1172
130	1259

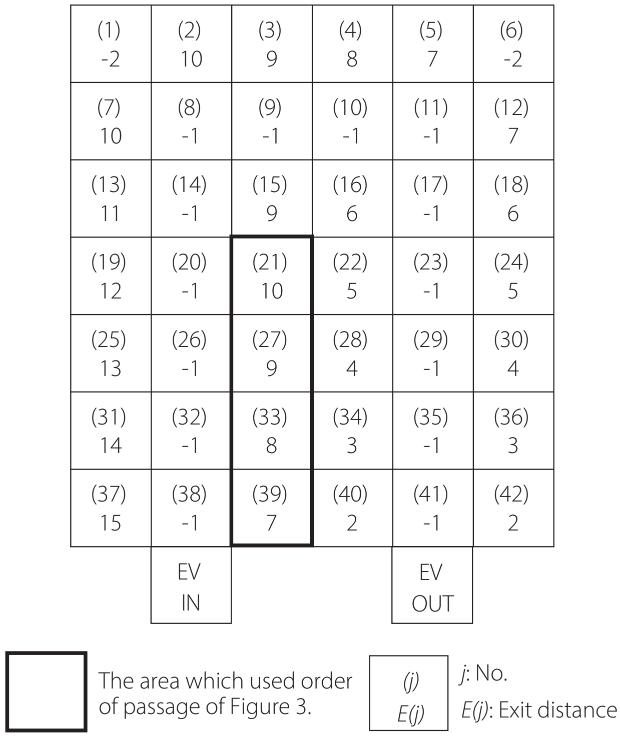
Notes: *M** = Number of parking cars; Using extended rand() in uniform random number.

Table 8: *T* when *m = 4, U = 42, S = 26, γ = 1* (Experiment 4 of using Xorshift)

*M**	Total of exit distance for all parking cars*(T)*
10	33
20	90
30	158
40	248
50	332
60	442
70	453
80	564
90	699
100	839
110	916
120	1052
130	1191

Notes: *M** = Number of parking cars; Using Xorshift in uniform random number.

regardless of the order of exit, when *M* = 130, T* is the total moving time from all basic unit spaces to the exit on the 1st basement floor. The combinations of 120 elementary unit spaces chosen from 130 elementary unit spaces happened to become equal. From Experiment 3 onwards, the trend is the same between using extended rand() and using Xorshift, although there are some numerical differences. When comparing the result (*T*) of Experiment 3 and the results (*T*) of Experiments 4 and 5, the results (*T*) of Experiments 4 and 5 are



The No.*j* of unit space is passage, when the term *E(j)* is –1.
The No.*j* of unit space is complex unit space, when the term *E(j)* is –2.

Figure14: Unit space No. on 1st basement floor in Figure 1 and exit distance of the basic unit space using procedure for creating a passage in Figure 3

smaller. So, we can see that Type 2 and 3 auto valet parking are more efficient than Type 1. When comparing the result of Experiment 4 and the result of Experiment 5, the result of Experiment 5 is improved by about 10 % in the case of using extended rand() at max of *M** (number of parking cars). However, the result of Experiment 5 is improved by only about 4 % in the case of using Xorshift. Also, while *M** is small, *T* may be smaller in the result of Experiment 4. Therefore, although Type 3 auto valet parking is an improved version of Type 2 auto valet parking, it can be seen that Type 2 auto valet parking is sometimes more efficient when the number of parking spaces in a multi-story car park is small. The reason for this is that the area of the unit space using the passage generation procedure of Figure 3 was scarcely located near the exit, or such a unit space was rarely selected by uniformly distributed random numbers. Furthermore, it is thought that the reversal occurs because a unit space far from the exit is selected. A comparison of the results of Experiment 4 and Experiment 5 will be made in a little more detail. In both experiments, uniform distribution random numbers are used for the order of exit operation, but the basic principle is first-in first-out. In other words, it is considered to be basic to exit in the order of arrival. In that case, the time *(T)* to exit the parking lot will be

Table 9: *T* when *m = 4, U = 42, S = 26, γ = 1* (Experiment 5 of using extended rand())

*M**	Total of exit distance for all parking cars*(T)*
10	32
20	96
30	150
40	232
50	330
60	382
70	442
80	531
90	669
100	779
110	904
120	1035
130	1158

Notes: *M** = Number of parking cars; Using extended rand() in uniform random number.

Table10: *T* when *m = 4, U = 42, S = 26, γ = 1* (Experiment 5 of using Xorshift)

*M**	Total of exit distance for all parking cars*(T)*
10	34
20	93
30	166
40	237
50	310
60	401
70	507
80	599
90	691
100	804
110	934
120	1009
130	1144

Notes: *M** = Number of parking cars; Using Xorshift in uniform random number.

*2M** for both Type 2 and Type 3 auto valet parking. Also, the rate of improvement of the exit system for Type 3 auto valet parking depends on the area of the unit space using procedure for creating a passage in Figure 3. Therefore, a method to increase this area is required.

Experiments 3, 4, and 5 assume that the speed of the automatic pallet is constant and the movement time is proportional to the number of unit spaces moved. Also, the elevator movement time is proportional to the number of floors

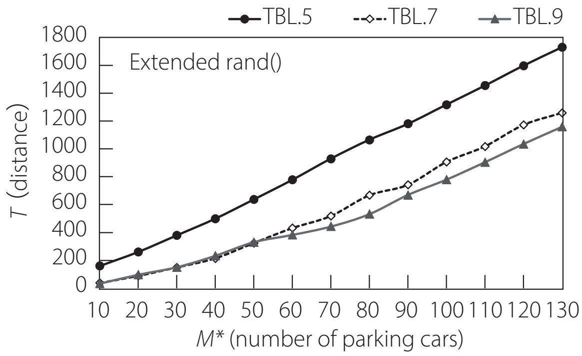


Figure15: Comparative diagram plotting the data in Tables 5, 7 and 9 (Using extended rand())

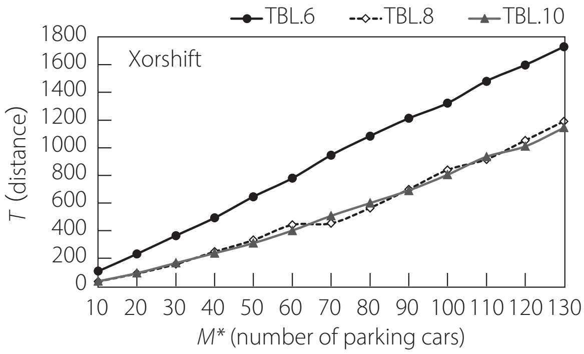


Figure16: Comparative diagram plotting the data in Tables 6, 8 and 10 (Using Xorshift)

moved, and for convenience it is assumed that the single unit space movement time is the same as the elevator movement time per floor. Therefore, from the experimental results, it was confirmed that the exit system for automated valet parking proposed by Funase et al. (2022c; 2022d) is effective in improving exit time efficiency.

Based on these results, we were able to propose a model for automated valet parking with good exit efficiency. However, in the actual field, various cases occur, and it is desirable to be able to deal with them as much as possible. Specifically, when the parking space is full, it is possible to temporarily use the passage unit space, which is a future research topic.

References

Autonomous Driving Lab (2020). Mechanism of automated valet parking system and the data shared with the system (original in Japanese: Jido valet parking no shikumi ya yaritori sareru data ha?) (Retrieved June 30, 2022 from https://jidounten-lab.com/u_parking-34).

Funase, S., Shimauchi, T., Kimura, H., and Nambo, H. (2022a). Proposal of a multi-story car park with cell pallets to maximize space utilization efficiency in confined area. *International Journal of Engineering & Technical Research*, Vol. 12, No. 1, 1-5.

Funase, S., Shimauchi, T., Kimura, H., and Nambo, H. (2022b).

A proposal of automated valet parking system using automatic pallet. *Studies in Science and Technology*, Vol. 11, No. 2, 147-150.

Funase, S., Shimauchi, T., Kimura, H., and Nambo, H. (2022c). A proposal of parking location determination method to realize high time-efficient exit procedures in automated valet parking using automated pallets. *International Journal of Engineering & Technical Research*, Vol. 12, No. 2, 1-8.

Funase, S., Kimura, H., and Nambo, H. (2022d). About how to move automatic pallets to improve the time efficiency of exit operation in auto valet parking. (posting)

Japan Automobile Research Institute (2021). Research and development on autonomous driving system (Original in Japanese: Jido soukou sisutemu no kenkyuu kaihatsu) (Retrieved June 30, 2022 from https://www.jari.or.jp/research-content/mobility/133).

Kimura, H. and Oyabu, T. (1989). *Introduction to information science*. Kyoritsu Shuppan.

Kishida, K. and Fujii, R. (1974). *Simulation exercise*. Sanpou Shuppan.

Kitagawa Corporation (2021). A new drive-in multi-story parking: Super long-span system (Original in Japanese: Shingata Jisoushiki Rittai Chusyajyou Supa Longu Supan Taipu) (Retrieved March 20, 2022 from https://prod.kiw.co.jp/parking/relation/2741.html).

Kobayashi, K. (2022). pseudorandom number generator (Retrieved January 8, 2023 from http://kobayashi.hub.hit-u.ac.jp/topics/rand.html).

Ministry of Land, Infrastructure, Transport and Tourism (2019). Strategic conference for the advancement of utilizing public and private sector data, strategic headquarters for the advanced information and telecommunications network society, "Public-Private ITS Initiavie/Roadmaps 2019 (Retrieved June 30, 2022 from https://japan.kantei.go.jp/policy/it/2019/2019_roadmaps.pdf).

Saito Y. (2018). Automated valet parking system: Is it profitable? Can the sytem function smooth enough to overcome users' preference to park by themselves (Original in Japanese: Valet Parking de Chusya-jyo ha moukaru ka? "jibun de yaru hou ga hayai" ha koeraeru ka?). *MONOist* (Retrieved June 30, 2022 from https://monoist.itmedia.co.jp/mn/articles/1811/14/news056_2.html).

Tanikawa, H. (2019). Current status and challenges for realizing automated valet parking (original in Japanese: Jido Valet Parking Jitsugen ni muketa genjyo to kadai) (Retrieved June 30, 2022 from http://japan-pa.or.jp/20190208/4294).

Yamazaki, A., Izumi, Y., Yamane, K., Nomura, T., and Seike, Y. (2019). Development of control technology for controlling automated valet parking. *Denso Ten Technical Review*, Vol. 3, 18-23.

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